The quantum state of the multiverse

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Salvador Robles-Pérez
Relativity and Cosmology Group
IFF – CSIC

Outline

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    - 3.2. 2nd-quantization: the wavefunction of the universe.
    - 3.3. 3rd-quantization: the wavefunction of the multiverse.
    - 3.4. Parent and baby universes: the space-time foam.
4. Conclusions.

1. Motivation to study the quantum multiverse.

- The multiverse is an idea which appears in many different cosmological scenarios:
  - Everett and post-Everett quantum formulations.
  - Chaotic inflation.
  - Landscape in string theory.
  - Smolin multiverse.
  - Ekpirotic model (branes).
  - Dark and phantom energy models of multiverse.
- The multiverse of “branches” is inherent to a quantum theory, by the principle of superposition, in any consistent interpretation of quantum cosmology.
- Suggestions for the quantum nature of the universe.

2. Physics or metaphysics?

- The idea of the multiverse arises in a natural way in many cosmological models.
- However, might the effects of the multiverse be observed?
    - Modifications of current semi-classical predictions:
        - cosmic radiation,
        - gravitational waves.
    - Vacuum state of gravity.
    - Vacuum state of matter fields.
    - Cosmic entanglement and quantum communication channels.
    - Superluminal travels and classical communication channels.

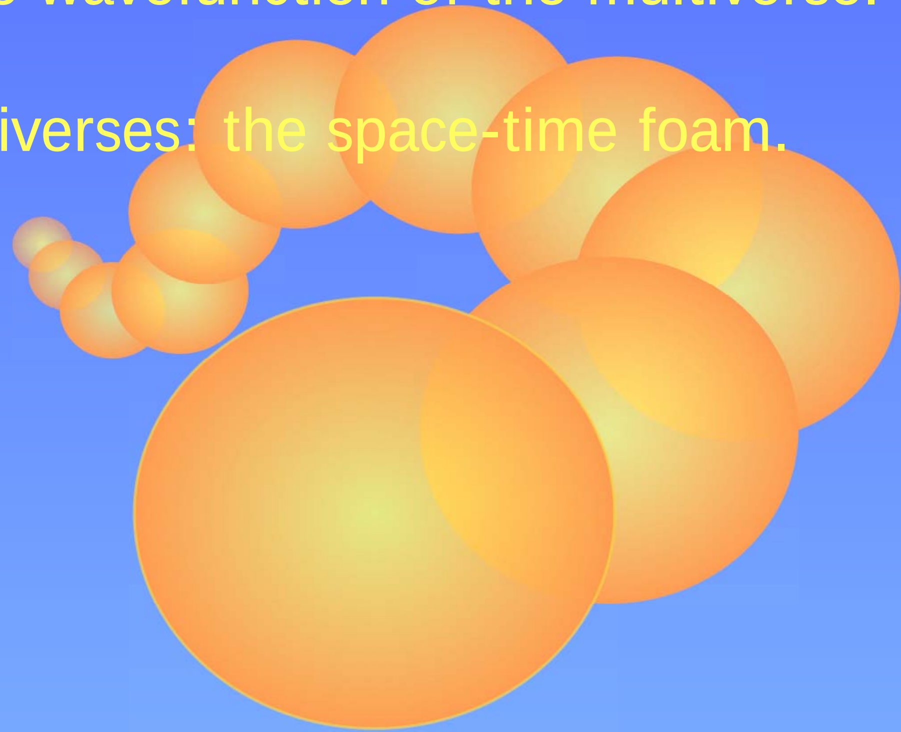
3. The quantum state of the multiverse

3.1. Classical single universes.

3.2. 2nd-quantization: the wavefunction of the universe.

3.3. 3rd-quantization: the wavefunction of the multiverse.

3.4. Parent and baby universes: the space-time foam.



3.1. Classical single universes.

- Let us consider a flat FLRW space-time:

$$ds^2 = -N^2(t)\,dt^2 + a^2(t)\,d\Omega_3^2(r,\theta,\varphi)$$

$N(t)$: lapse function.
$a(t)$: scale factor.

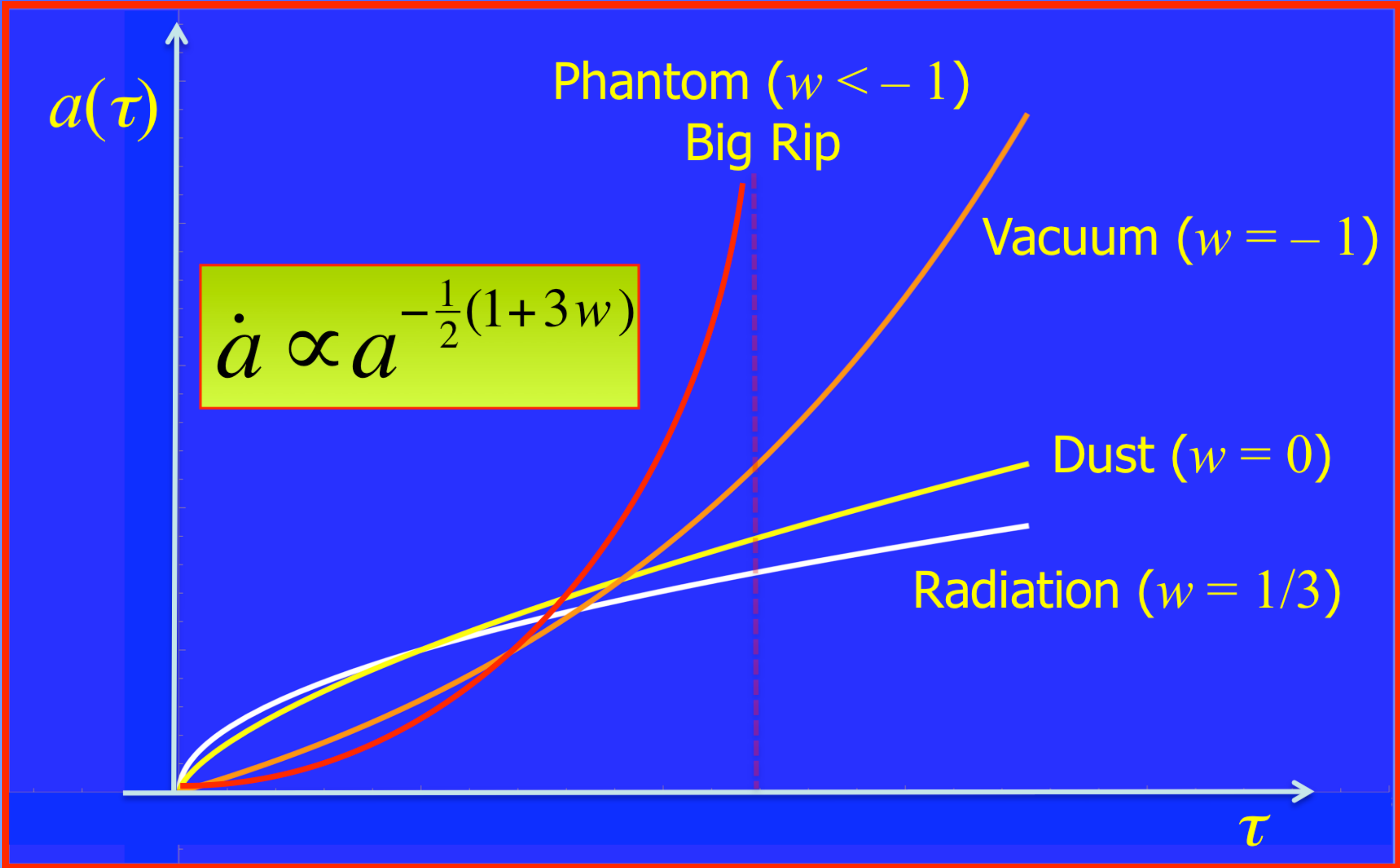
- Filled with a fluid described by an equation of state, $p = w\,\rho$, being w a constant:

$$d\rho = -3(p+\rho)\frac{da}{a} \quad \Rightarrow \quad \rho(a) \propto a^{-3(1+w)}$$

(Cosmic energy conservation)

3.1. Classical single universes: time evolution.

- With respect to the proper time, τ $(N = 1)$:



Time evolution of the scale factor for a flat FLRW universe filled with a fluid described by the equation of state, $p = w\,\rho$, being w = const.

3.2. 2nd-quantization: the wavefunction of the universe

- Canonical Dirac's quantization:

Friedmann equation

$$H(a,p) = -\frac{2\pi G}{3}\frac{p^2}{a} + \rho_0 a^{-3w} = 0$$

Wheeler-De Witt equation

$$\hat{H}(a,-i\hbar\frac{\partial}{\partial a})\psi_0(a) = 0$$

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Wheeler-De Witt equation

$$\hat{H}(a, -i\hbar \frac{\partial}{\partial a}) \psi_0(a) = 0$$

- The 2nd-quantized wave function of the universe, $\psi_0(a)$, is the solution of the Wheeler-De Witt equation:

$$\dot{\psi} \equiv \frac{\partial \psi}{\partial a}$$

$$\ddot{\psi}_0 + \lambda_0^2 \, a^{-3w+1} \psi_0 = 0$$

$$\begin{cases} \psi_{01}(a) = \sqrt{a} \; J_{\frac{1}{2q}}(\lambda a^q) \\ \psi_{02}(a) = \sqrt{a} \; Y_{\frac{1}{2q}}(\lambda a^q) \end{cases}$$

$$\lambda = \frac{\lambda_0}{q}$$

$$q = \frac{3}{2}(1 - w)$$

J_n, Y_n : Bessel functions of first and second kind and order n.

3.3. 3rd-quantization.

- However, in order to properly represent the quantum multiverse, we have to turn to a 3rd-quantization procedure, in which creation and annihilation operators (of universes) can be well-defined:

Gravity (space-time)	**Fields defined upon space-time**
2nd-quantization $\psi(a\,[,\phi_1,\phi_2,\ldots])$	Quantum mechanics $\phi(x,t)$
3rd-quantization $\Psi(\psi,a), b_\psi^+, b_\psi$	2nd-quantization $\Psi(\phi,x), a_\phi^+, a_\phi$

3.3. 3rd-quantization: Hamiltonian formalism.

- The 2nd-quantized Wheeler-De Witt can be considered as the equation which describes a "classical" harmonic oscillator:

$$\ddot{\psi}_0 + \lambda_0^2 \, a^{-3w+1} \psi_0 = 0 \qquad \left(\ddot{\psi} \equiv \frac{\partial^2 \psi}{\partial a^2} \right)$$

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- It can be further quantized (3rd-quantization), with the Hamiltonian of a quantum harmonic oscillator with "time"-dependent frequency:

$$\mathrm{H}_{\mathrm{h.o.}} = \frac{1}{2} p_\psi^2 + \frac{\omega^2(a)}{2} \psi^2 \qquad \begin{cases} \omega(a) = \lambda_0 \, a^{q-1}, \\ \lambda_0 = \dfrac{1}{\hbar} \sqrt{\dfrac{3\rho_0}{2\pi G}} \end{cases}$$

3.3. 3rd-quantization: the wavefunction of the multiverse.

- The 3rd-quantized wavefunction is given then by the solutions of the (3rd-quantized) Schrödinger equation for the harmonic oscillator, with scale factor-dependent frequency:

$$\hat{\mathrm{H}}_{\mathrm{h.o.}}\,\Psi(\psi,a) = -i\hbar\frac{\partial}{\partial a}\Psi(\psi,a)$$

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- It can be interpreted as the wavefunction of the multiverse:

$$\left|\Psi\right\rangle = \sum_n \Psi_n(a)\left|n\right\rangle \qquad \hat{H}\left|n\right\rangle = \hbar\omega\left(n+\tfrac{1}{2}\right)\left|n\right\rangle$$

$\Psi_n(a)$ is the probability amplitude of *n* universes at "*time a*", or the probability amplitude of *n* universes with the scale factor *a*.

3.3. 3rd-quantization: the wavefunction of the multiverse.

- The frequency is determined by the potential of the 2nd-quantized theory, i.e., by the kind of energy-matter content considered in each universe of the whole multiverse.

fluid	$w = p/\rho$	$\omega^2(a)$
radiation	1/3	λ^2_0
dust	0	$\lambda^2_0\, a$
"gravitational"	-1/3	$\lambda^2_0\, a^2$
quintessence	-2/3	$\lambda^2_0\, a^3$
Λ	-1	$\lambda^2_0\, a^4$

$$\omega(a) = \lambda_0\, a^{q-1}$$

$$q = \frac{3}{2}(1-w)$$

3.3. 3rd-quantization: the wavefunction of the multiverse.

- There is a unitary relation between the quantum states of an harmonic oscillator with time-dependent frequency and those corresponding to the static case,

$$\left|\Psi_N\right\rangle = \frac{1}{\sqrt{R(a)}} U_\omega \left|\tilde{\Psi}_N\right\rangle$$

$$\tilde{H} = \frac{1}{2}\left(p_\varphi^2 + \varphi^2\right)$$

$$\tilde{H}\tilde{\Psi}_N = (N + \tfrac{1}{2})\tilde{\Psi}_N$$

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$$U_\omega(\psi, a) = e^{\frac{i}{2\hbar}\frac{\dot{R}}{R}\psi^2}$$

$$R(a) = \sqrt{\psi_{01}^2(a) + \psi_{02}^2(a)}$$

$$\psi_{01}(a) = \sqrt{\frac{\pi a}{2q}} J_{\frac{1}{2q}}(\omega a)$$

$$\psi_{02}(a) = \sqrt{\frac{\pi a}{2q}} Y_{\frac{1}{2q}}(\omega a)$$

3.3. 3rd-quantization: the wavefunction of the multiverse.

- The wavefunction of the multiverse can be written then as:

$$\Psi(\psi,a) = \sum_n C_n e^{i\alpha_n(a)} \left(\frac{1}{\sqrt{\pi\hbar}\, 2^n\, n! R} \right)^{\frac{1}{2}} e^{\frac{i}{2\hbar}\left(\frac{\dot{R}}{R} + \frac{i}{R^2}\right)\psi^2} \mathrm{H}_n\left(\frac{\psi}{R\sqrt{\hbar}}\right)$$

$$\alpha_n(a) = -\left(n + \tfrac{1}{2}\right) \int_0^a \frac{da'}{R^2(a)}$$

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$\mathrm{H}_n(x)$: Hermite polynomials.

- If different types of universes are considered:

$$\left|\Psi\right\rangle = \sum_{\vec{n}} \Psi_{\vec{n}}(\psi,a) \left| n_{1\omega_1}, n_{2\omega_2}, \cdots \right\rangle$$

$n_{i\omega_i}$, is the number of universes of type i, which are dominated by an energy-matter content represented by $\omega_i(a)$.

3.3. 3^{rd}-quantization: the wavefunction of the multiverse.

- The creation and annihilation operators, b^+ and b, turn out to be scale factor-dependent:

$$b(a) = \mu(a) b_0 + \nu(a) b_0^+ ,$$
$$b^+(a) = \mu^*(a) b_0^+ + \nu^*(a) b_0$$

b_0, b_0^+, creation and annihilation operators, with: $m = \omega = 1$

where:

$$\mu(a) = \frac{1}{2}\left(\frac{1}{R(a)} + R(a) - i\dot{R}(a)\right), \quad \upsilon(a) = \frac{1}{2}\left(\frac{1}{R(a)} - R(a) - i\dot{R}(a)\right)$$

$$|\mu(a)|^2 - |\upsilon(a)|^2 = 1$$

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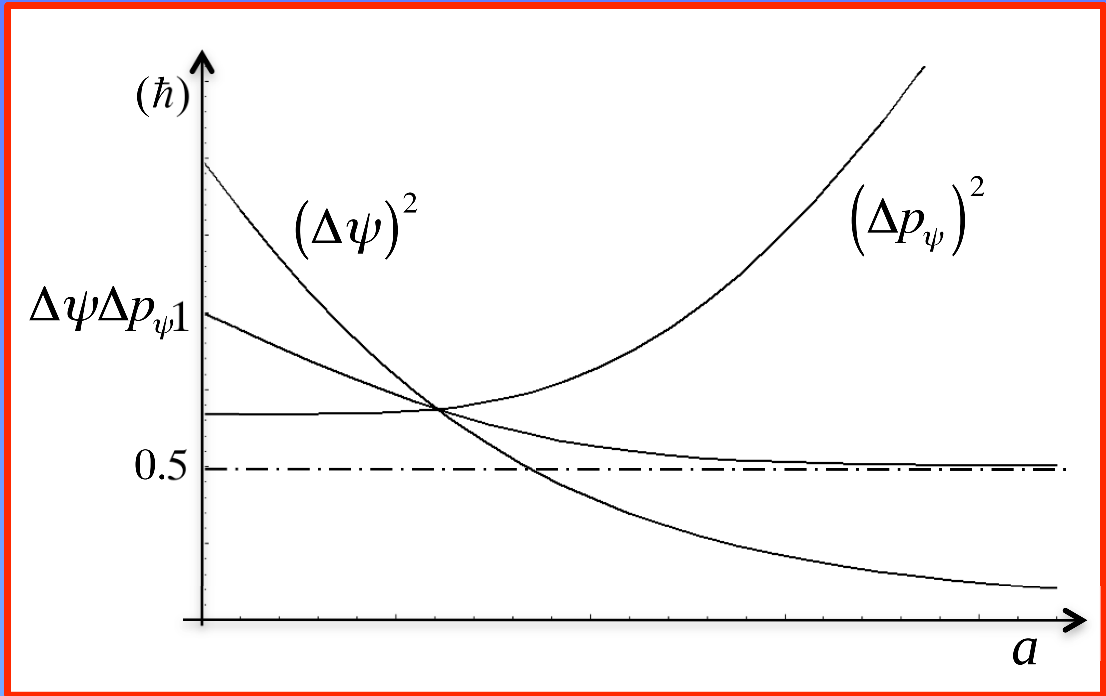
$$|\mu(a)|^2 - |\upsilon(a)|^2 = 1$$

The eigenstates of the annihilation operator (coherent states) turn out to be squeezed states (with no classical analog).

3.3. 3rd-quantization: the wavefunction of the multiverse.

- The uncertainties of the wavefunction of the universe, as a configuration variable of the wavefunction of the multiverse, and its conjugate momentum, are given by:

$$(\Delta\psi)^2 = \frac{\hbar}{2}|\mu - \nu|^2, \quad (\Delta p_\psi)^2 = \frac{\hbar}{2}|\mu + \nu|^2$$



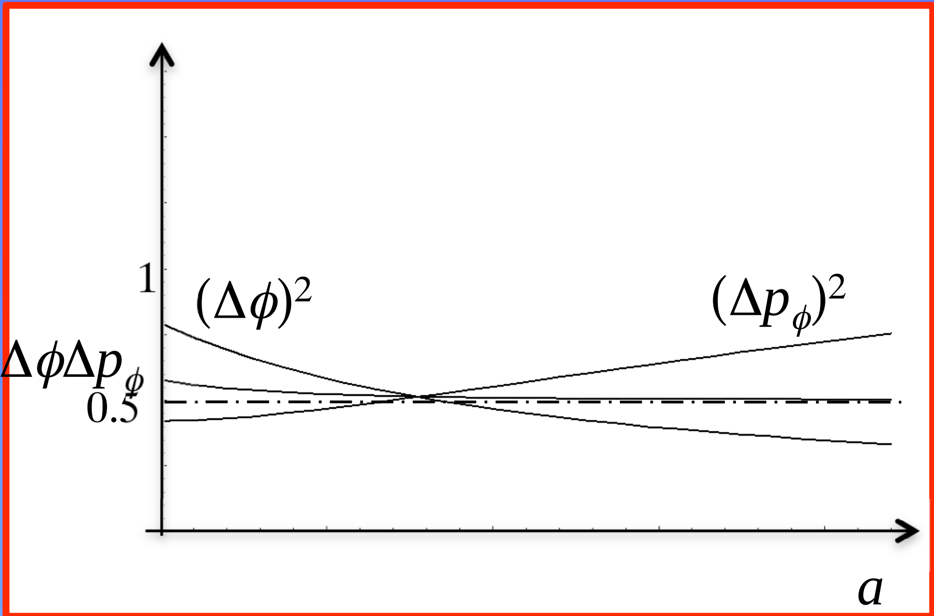
$w = -1$

(vacuum dominated)

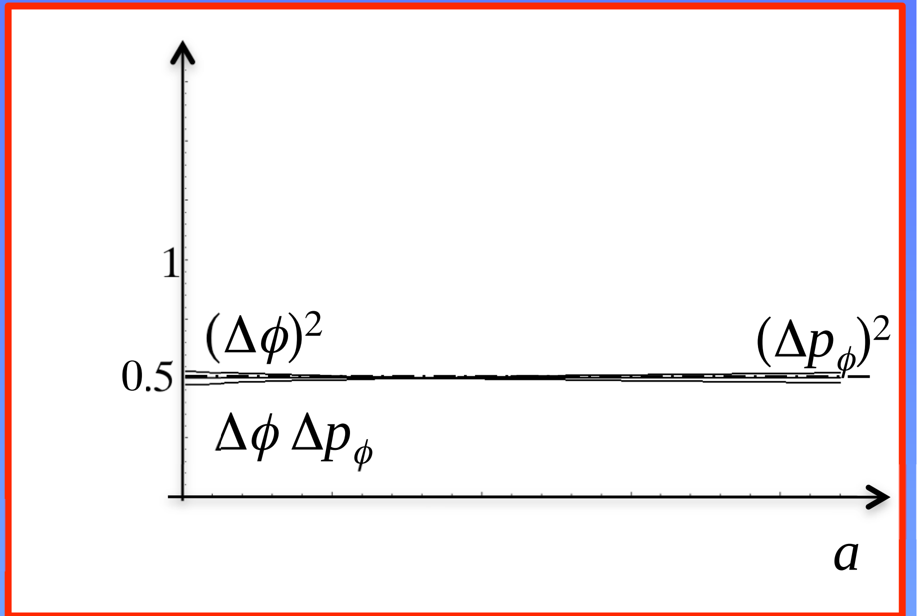
3.3. 3rd-quantization: the wavefunction of the multiverse.

- The squeezing effect (non-classical effect) is greater for accelerated universes, disappearing for a radiation dominated universe.

$w = 0$ (dust dominated)



$w = \frac{0.9}{3}$ (radiation dominated)



3.3. 3rd-quantization: the wavefunction of the multiverse.

- Therefore:

i) The quantum state of the multiverse turn out to be a squeezed state with no classical analog:

$$\Delta\psi\Delta p_{\psi} \rightarrow \frac{\hbar}{2} \quad (\text{at } a \rightarrow \infty), \text{with} : \Delta\psi \neq \Delta p_{\psi}$$

ii) For large values of the scale factor, the single universe approximation is valid:

$$\Delta\psi \rightarrow 0 \quad (\text{at } a \rightarrow \infty)$$

3.3. 3rd-quantization: the wavefunction of the multiverse.

- Therefore:

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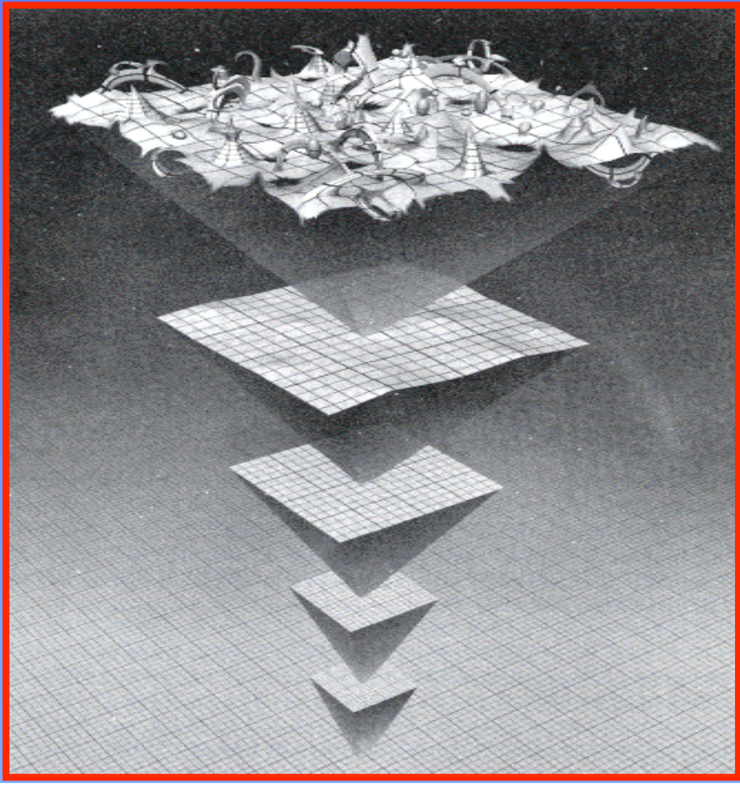
  ii) For large values of the scale factor, the single universe approximation is valid:

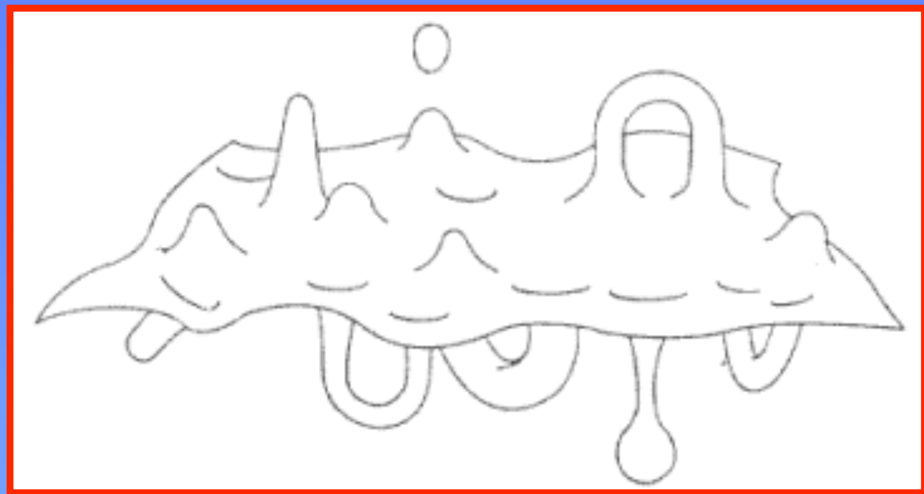
$$\Delta\psi \to 0 \quad (\text{at } a \to \infty)$$

- With the annihilation and creation operators, the many-universe system can represent either a multiverse made up of parent universes or a space-time foam (gravitational vacuum) of popping baby universes.

3.4. Parent and baby universes: the space-time foam.

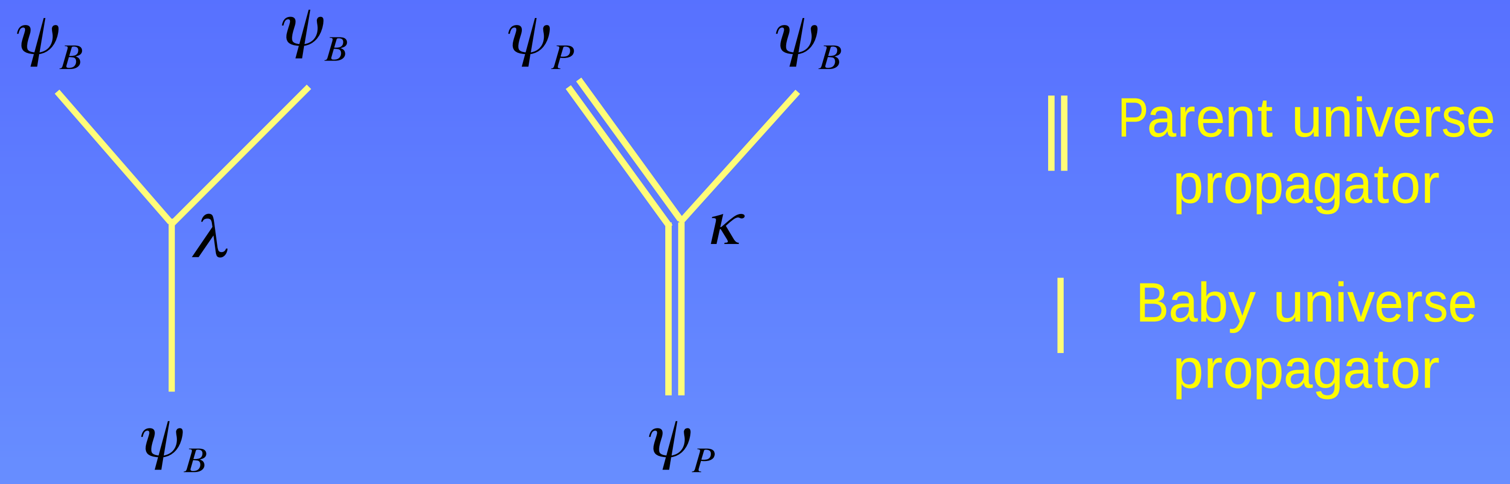
- At Planck length, the fluctuations of the geometry in the gravitational action are so large, that the vacuum turns out to be a quantum superposition of geometries, i.e., a “space-time foam” made up of baby universes (virtual closed universes of Planck length), blackholes and wormholes.





3.4. Parent and baby universes: the space-time foam.

- Nucleation of baby universes can also be depicted in Feynman-like diagrams:



$$S[\psi] = \int da\left(-\dot{\psi}_P^2 + \omega_P^2\psi_P^2 - \dot{\psi}_B^2 + \omega_B^2\psi_B^2 + \kappa\psi_P^2\psi_B + \tfrac{\lambda}{3}\psi_B^3\right)$$

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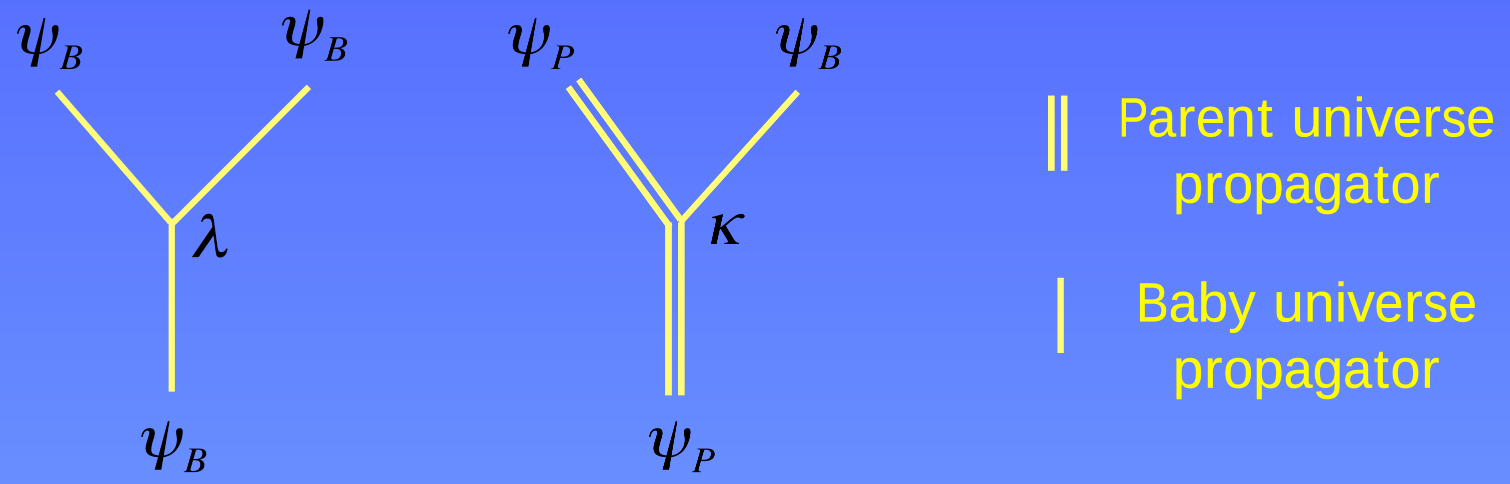

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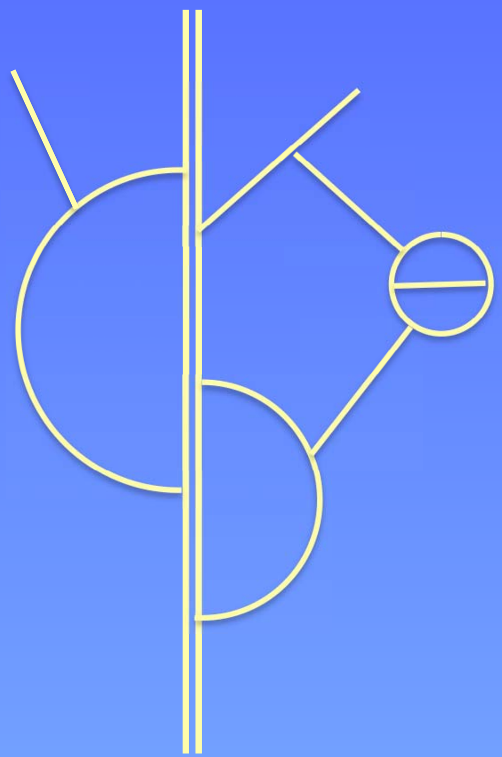
- There is a very large energy gap between parent and baby universes:

$$\omega_B \approx l_P^{q-1}, \omega_P \approx r_H^{q-1}$$

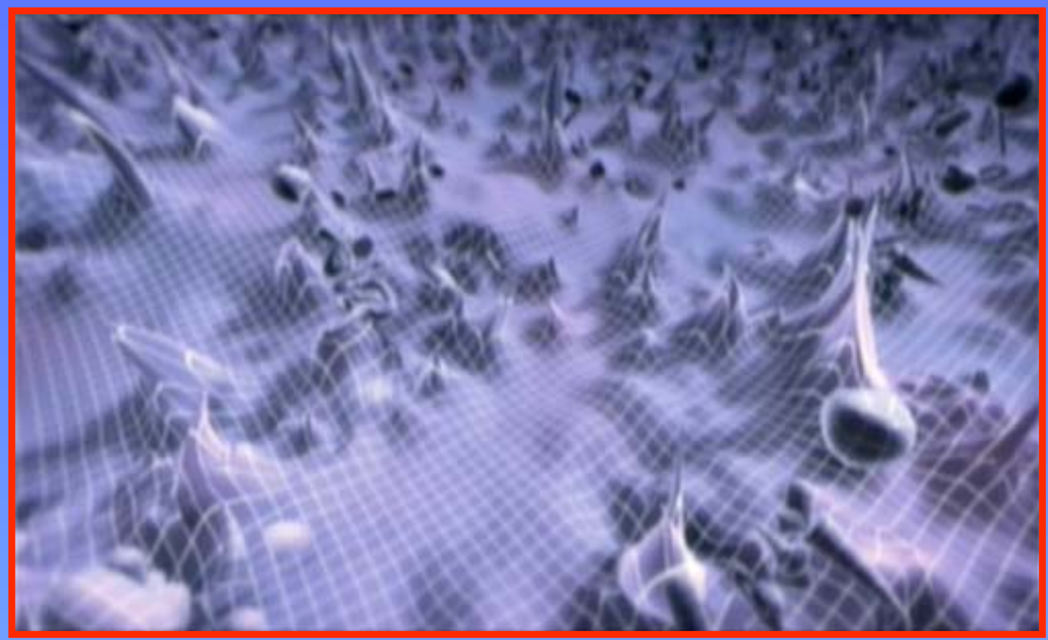
$r_H \approx$ Hubble scale

3.4. Parent and baby universes: the space-time foam.

- In the single universe approximation, a parent universe (of Hubble's length) propagates in a bath of baby universes.



Parent universe



3.4. Parent and baby universes: the space-time foam.

- Matter fields are coupled to the space-time foam (since they are propagating in space-time).
- In the single universe approximation, the coupling modify the ground state of matter fields and could provide us with a vacuum energy for the universe.
- A decoherence process has to be further analyzed in order to obtain the single universe approximation, and an hybrid action.
- In principle, the scale of the effects should be of Planck length, although it would depend on the decoherence process.

4. Conclusions

- A multiverse made up of homogeneous and isotropic universes can be quantum mechanically described by the wavefunction of an harmonic oscillator with scale factor-dependent frequency.

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- A multiverse made up of homogeneous and isotropic universes can be quantum mechanically described by the wavefunction of an harmonic oscillator with scale factor-dependent frequency.
- The quantum state of the multiverse turns out to be a squeezed state, having no classical analog (in the sense of Bell's inequalities)
- The squeezing effect (non-classical effect) is greater for a multiverse made up of accelerated universes.