First MCMC lessons on cosmography

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Work (in good progress) in collaboration with S. Capozziello and V. Salzano

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  - It is only at the interpretation stage that one examines the results in the light of a specific setup.

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  - The datasets used: SNIa and direct Hubble data

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- Using the series of $a(t)$ one finally gets a power-series of D in terms of z and the cosmographic parameters:

$$D(z) = \frac{cz}{H_0} \left\{ 1 + \mathcal{D}_z^1(q_0)\, z + \mathcal{D}_z^2(q_0, j_0)\, z^2 + \mathcal{D}_z^3(q_0, j_0, s_0)\, z^3 + \mathcal{O}[z^4] \right\}.$$

- The Hubble free *luminosity distance* in a spatially flat universe is given by:

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- But the SNIa luminosity data actually inform us about the *distance modulus*: $\mu(z) = 5 \log_{10} d_L(z) + \mu_0$
- Thus, we have to construct its series too.

- The distance modulus series is

$$\mu(z) = \frac{5}{\log 10} \cdot \left(\log z + \mathcal{M}^1 z + \mathcal{M}^2 z^2 + \mathcal{M}^3 z^3 + \mathcal{M}^4 z^4\right) + \mu_0 \, ,$$

with

$$\mathcal{M}^1 = -\frac{1}{2} \left[-1 + q_0\right] \, ,$$

$$\mathcal{M}^2 = -\frac{1}{24} \left[7 - 10 q_0 - 9 q_0^2 + 4 j_0\right] \, ,$$

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- For other distance definitions we have complicated expressions as well.

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- We compare results obtained from both definitions.

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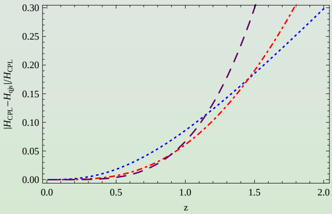
  - Not so obvious, it seems.
- A comparison with the popular CPL parametrization can shed some light:

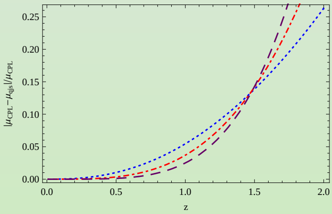
$$w = w_0 + w_1 z/(1+z)^{-1}.$$

$$\Omega_m \longrightarrow \Omega_m(q_0, j_0, s_0, l_0, \dots)$$
$$w_0 \longrightarrow w_0(q_0, j_0, s_0, l_0, \dots)$$
$$w_1 \longrightarrow w_1(q_0, j_0, s_0, l_0, \dots)$$

- The correspondence is:

$$
\begin{aligned}
q_0 &= \frac{1}{2} + \frac{3}{2}(1-\Omega_m) w_0 \\
j_0 &= 1 + \frac{3}{2}(1-\Omega_M)\left[3w_0(1+w_0) + w_1\right] \\
s_0 &= -\frac{7}{2} - \frac{33}{4}(1-\Omega_m) w_1 - \frac{9}{4}(1-\Omega_M)\left[9 + (7-\Omega_M) w_1\right] w_0 \\
&\quad -\frac{9}{4}(1-\Omega_M)(16-3\Omega_M) w_0^2 \frac{27}{4}(1-\Omega_M)(3-\Omega_M) w_0^3 \\
l_0 &= \frac{35}{2} + \frac{1-\Omega_m}{4}\left[213 + (7-\Omega_m) w_1\right] w_1 + \\
&\quad + \frac{(1-\Omega_m)}{4}\left[489 + 9(82-21\Omega_m) w_1\right] w_0 + \\
&\quad + \frac{9}{2}(1-\Omega_m)\left[67 - 21\Omega_m + \frac{3}{2}(23-11\Omega_m) w_1\right] w_0^2 + \\
&\quad + \frac{27}{4}(1-\Omega_m)(47-24\Omega_m) w_0^3 + \frac{81}{2}(1-\Omega_m)(3-2\Omega_m) w_0^4
\end{aligned}
$$





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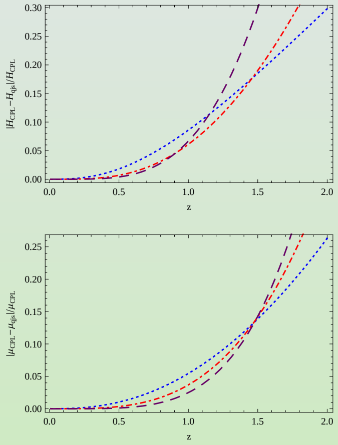

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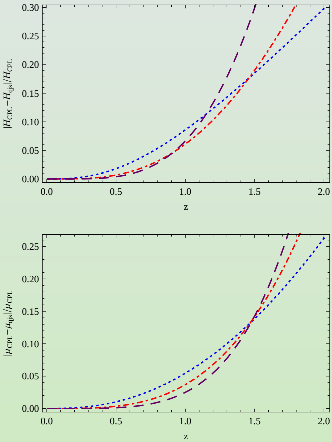

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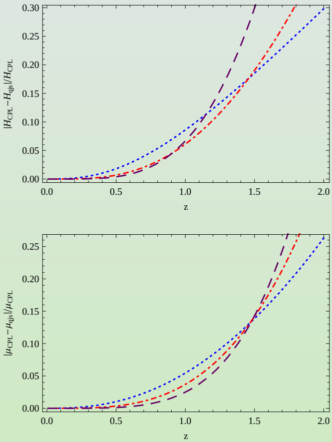

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- The 3rd order series seem to be the best compromise.

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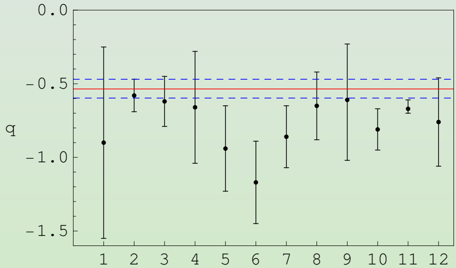
- Obtained with positive d_L and H^2 in $0 < z < 1.56$, using SN only and z-redshift expressions.

Our results and the literature

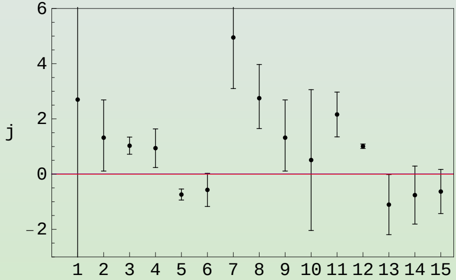


(1) - John (2004); (2) - Cattoen (2007), Legacy SN sample with z; (3) - Cattoen (2007), Gold SN sample with z;

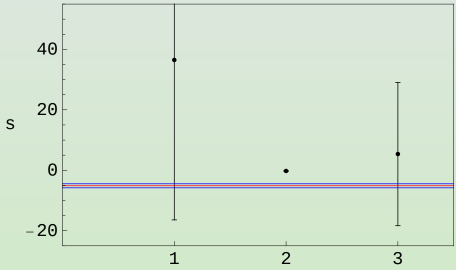
(4) - Cattoen (2007), Legacy SN sample with y; (5) - Cattoen (2007), Gold SN sample with y;

(6) - Rapetti (2006), HST; (7) - Rapetti (2006), Gold SN sample; (8) - Rapetti (2006), Legacy SN sample;

(9) - Rapetti (2006), X-ray clusters; (10) - Rapetti (2006), all subsamples; (11) - Poplawski (2006);



(1) - John (2004); (2) - Astier (2006); (3) - Cattoen (2007), Gold SN sample - mean; (4) - Cattoen (2007), Legacy SN sample - mean; (5) - Cattoen (2007), Gold SN sample with y; (6) - Cattoen (2007), Legacy SN sample with y; (7) - Rapetti (2006), HST; (8) - Rapetti (2006), Gold SN sample; (9) - Rapetti (2006), Legacy SN sample; (10) - Rapetti (2006), X-ray clusters; (11) - Rapetti (2006), all subsamples; (12) - Poplawski (2006); (13) - Capozziello, Izzo (2008); (14) - Capozziello, Izzo (2008); (15) - Capozziello, Izzo (2008)



(1) - John (2004); (2) - Poplawski (2006); (3) - Capozziello, Izzo (2008).

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- **Suggestions made by Ibericos participants**