HOLOGRAPHIC MULTIVERSE

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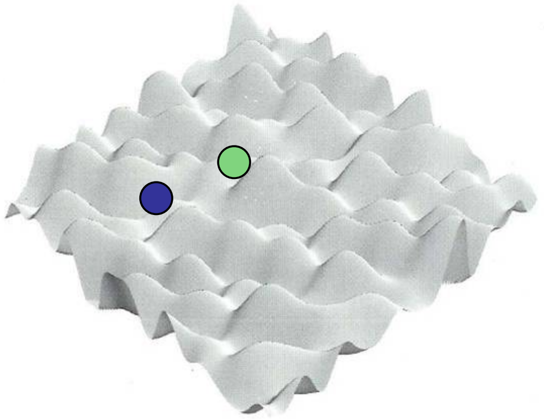
(with A. Vilenkin)

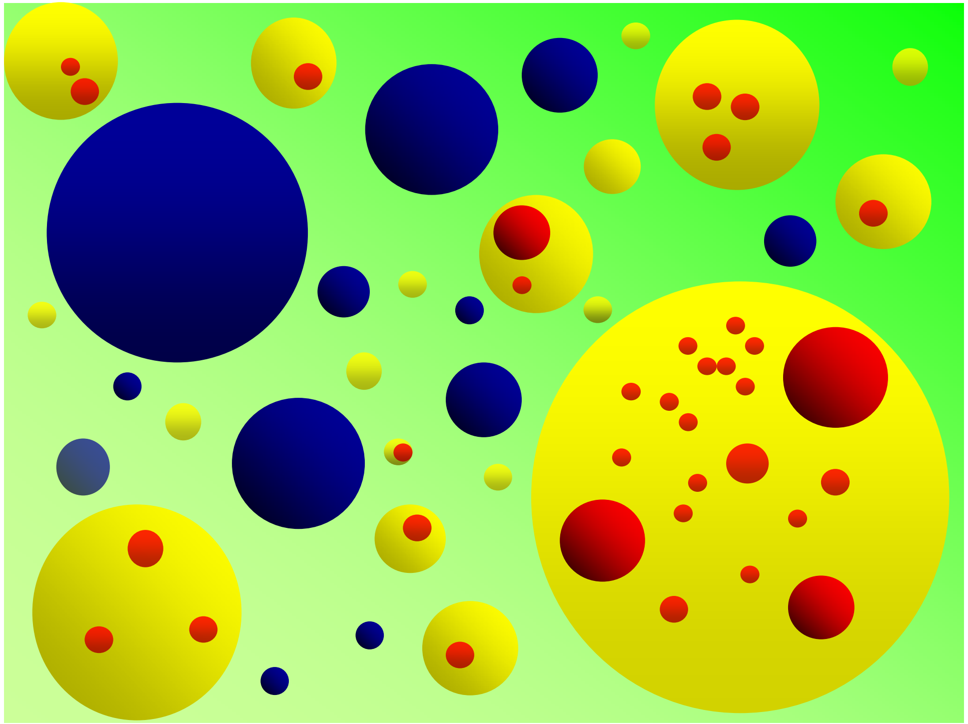
ICCUB

IBERICOS09

Eternally inflating multiverse

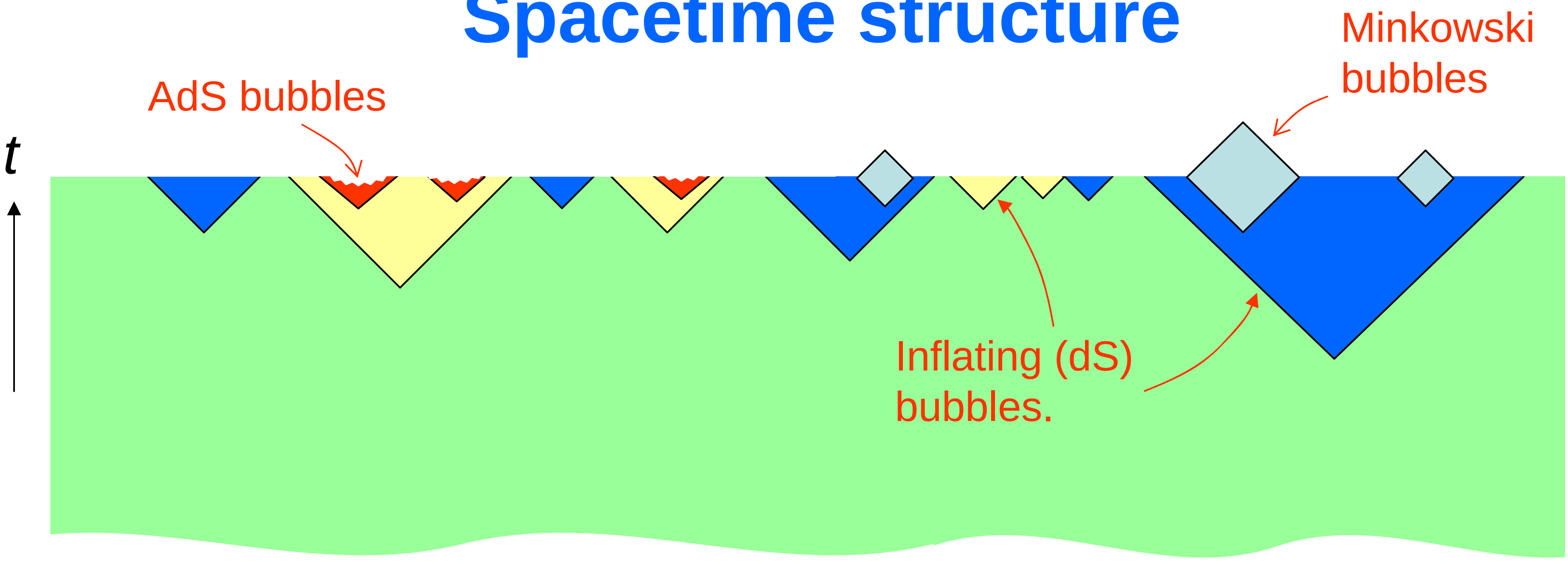


Field space



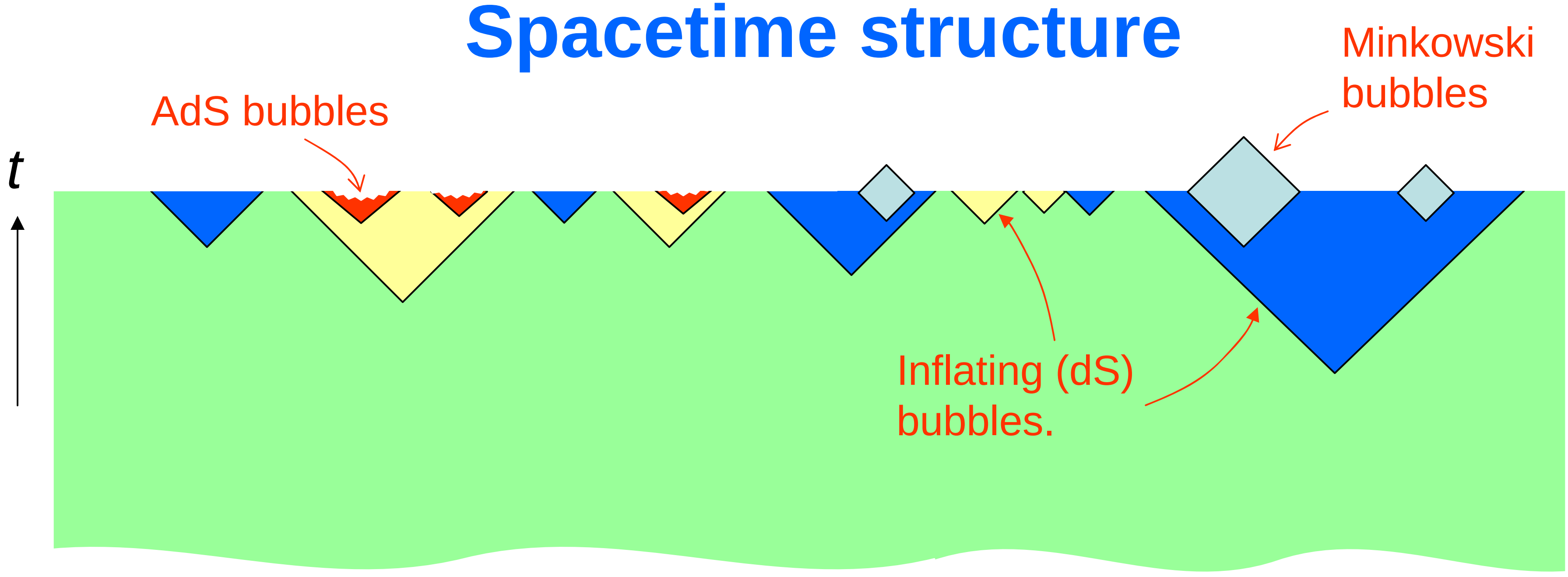
Physical space

Spacetime structure



- Bubbles nucleate and expand at nearly the speed of light.
- dS, AdS, and Minkowski bubbles

Spacetime structure

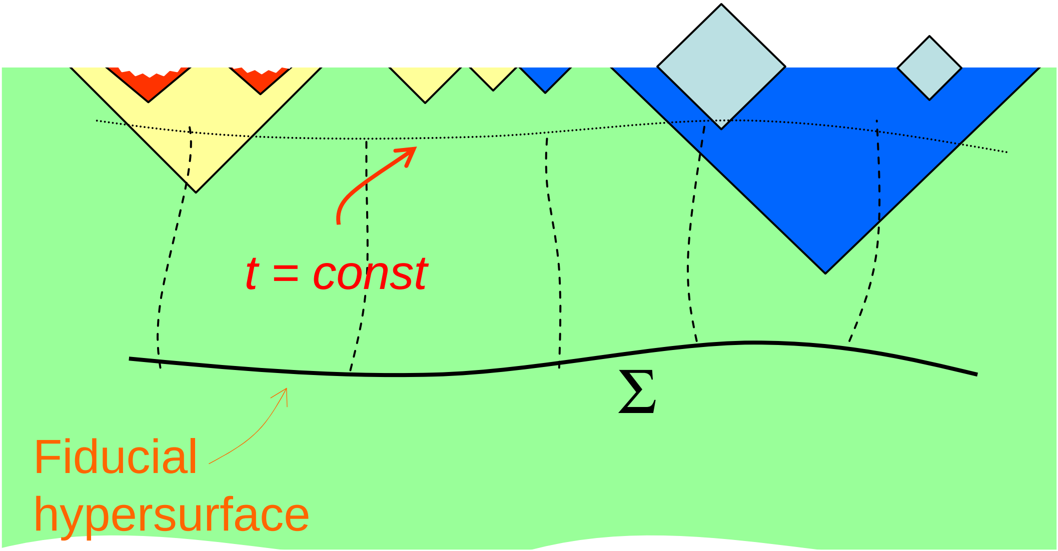


Everything that can happen will happen an infinite number of times. We have to learn how to compare these infinities. (Otherwise we cannot distinguish probable events from highly improbable & cannot make any predictions.)

Need a cutoff. Results are strongly cutoff-dependent.

Global time cutoff:

Count only observations that were made before some time *t*.



Garcia-Bellido, Linde & Linde (1994); Vilenkin (1995)

Possible choices of t :

(i) proper time $t = \tau$ along geodesics orthogonal to Σ ;

(ii) scale-factor time, $t = \ln a$.

$t \to \infty$ ⟹ steady-state evolution.

The distribution does not depend on the choice of Σ
-- but depends on what we use as *t*.

Attractor distribution:

Fraction of volume $V_i(t)$ in inflating vacuum of type I

$$dt = H^{\alpha}\, d\tau$$

$\alpha = 0$ Proper time gauge

$\alpha = 1$ Scale factor gauge $t = \log a$

$$\boxed{\frac{dV_i}{dt} = M_{ij} V_j + 3 H_i^{1-\alpha} V_i}$$

rate equation

$$M_{ij} = \underbrace{\lambda_{ij}}_{\text{Gained from other vacua}} - \underbrace{\delta_{ij} \sum_r \lambda_{ri}}_{\text{Lost to other vacua}}$$

$$\lambda_{ij} = \frac{4\pi}{3} H_j^{-3-\alpha}\, \Gamma_{ij}$$

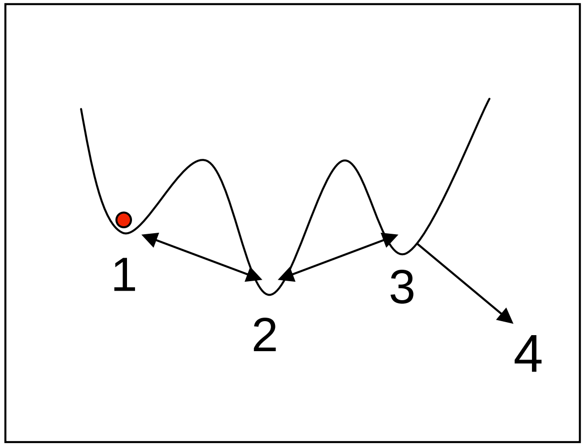
From bubbles of type "i" in vacuum "j".

To each irreducible ``landscape" there corresponds an attractor volume distribution.

$$V_i(a) \to V_i^{(0)}\ a^{3-q} \qquad 0 < q \ll 1$$

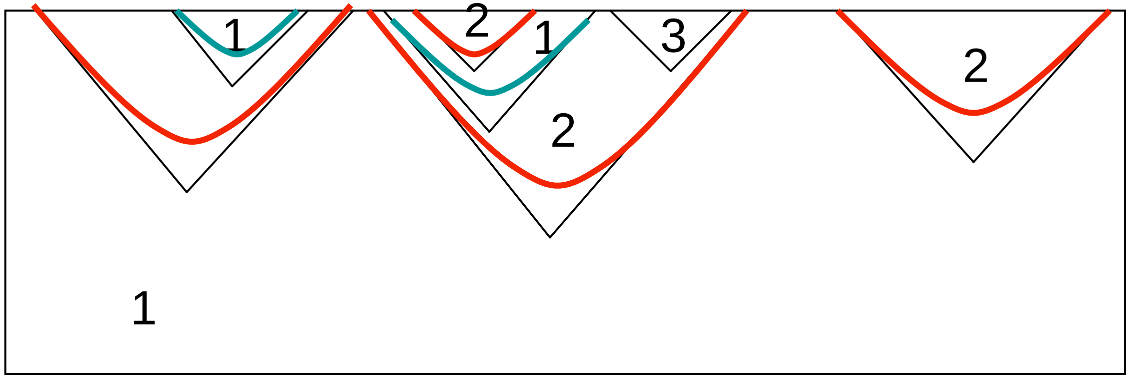
In this sense, initial conditions do not play a role.

THEORY



J.G., Schwartz-Perlov, Vilenkin & Winitzki (2005)

(Self-similar fractal)



MULTIVERSE

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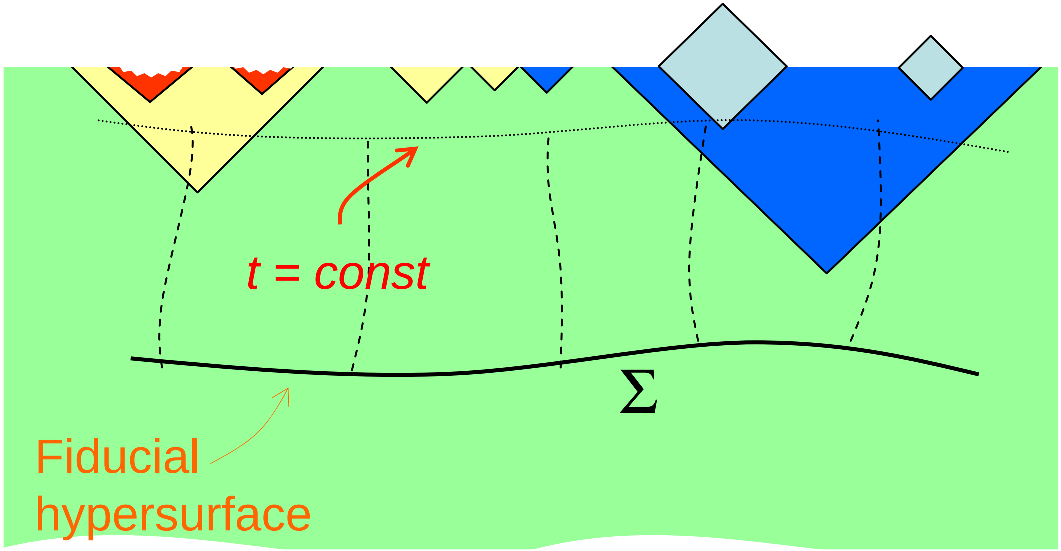

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-- but depends on what we use as *t*.

Measure proposals:

- Proper time cutoff — ***Garcia-Bellido, Linde & Linde (1994)***; ***Vilenkin (1995)***
- Scale factor cutoff — ***Garcia-Bellido, Linde & Linde (1994)***; ***De Simone, Guth, Salem & Vilenkin (2008)***
- Stationary — ***Linde (2007)***
- Pocket-based — ***J.G., Schwartz-Perlov, Vilenkin & Winitzki (2005)***; ***Easther, Lim & Martin (2005)***
- Causal-patch — ***Bousso (2006); Susskind (2007)***

Empirical approach:

Investigate different measure proposals and discard those which suffer from internal inconsistencies or strongly disagree with observations.

	Youngness paradox	Q catastrophe	Dependence on initial state
Proper time cutoff			
 **Scale factor cutoff**			
Pocket-based measure			
Stationary measure			
Causal patch measure			

This talk:

A measure from fundamental theory

Based on work with Alex Vilenkin

- The dynamics of the multiverse may be encoded in its future boundary (suitably defined).

  Inspired by holographic ideas: *Quantum dynamics of a spacetime region is describable by a boundary theory.*

- The measure can be obtained by imposing a UV cutoff in the boundary theory.
  Related to scale-factor cutoff.

Holography

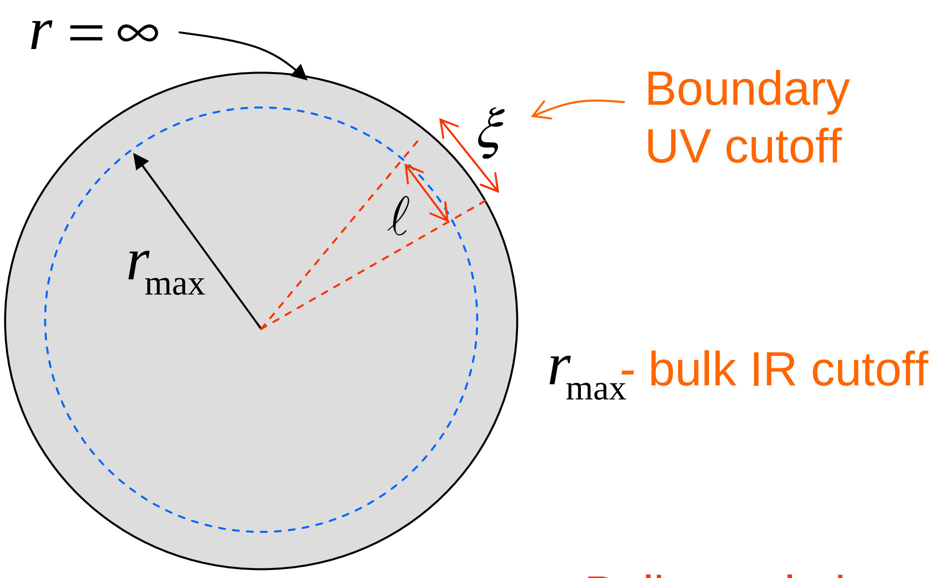
Maldacena (1998)
Strominger (2001)
Freivogel, Sekino,
Susskind & Yeh (2006)

AdS_{d+1}/CFT_d correspondence

Maldacena (1998)

Euclidean AdS:

$$ds^2 = dr^2 + \sinh^2 r \, d\Omega_D^2$$



Regulate boundary theory:

Integrate out short-wavelength modes of wavelength $\lambda_B < \xi$.

The corresponding bulk modes have minimum wavelength

$$\lambda_{\min}(r) = \xi \sinh r.$$

Require $\lambda_{\min} < \ell$ — Bulk resolution scale

$$\Rightarrow \quad \sinh r_{\max} = \ell / \xi$$

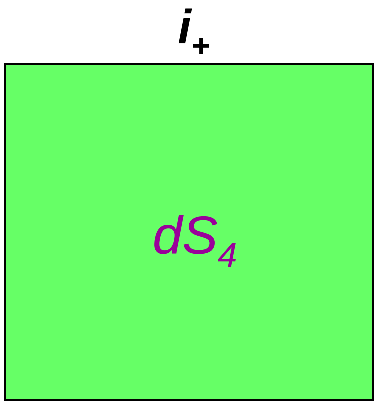
$$r_{\max} \to \infty \quad \Leftrightarrow \quad \xi \to 0.$$

Variation of r_{max} $\Leftrightarrow$ RG flow in the boundary theory.

dS/CFT correspondence

Strominger (2001)

The 4D theory describing an asymptotically de Sitter space is equivalent to a 3D Euclidean CFT at future infinity ***i*₊**.



$$ds^2 = -dt^2 + H^{-2}\cosh^2(Ht)\, d\Omega_3^2$$

Future infinity is $S_3: \; t \to \infty.$

Some potential problems:

1- Scaling dimensions are complex for bulk fields with

$$m^2 > \left(\frac{d}{2}\right)^2 H^2$$

(these fields oscillate in time)

2- In String Theory, dS space is metastable, so there is no such thing as asymptotically dS space.

dS/CFT and the wave function of the universe

$$\varphi(t \to \infty) = \bar{\varphi}(\mathrm{x}^{\mathrm{i}})$$

$$\begin{cases} Z_{d+1}^{\mathrm{Bulk}}[\bar{\varphi}] = \int D\varphi\, e^{i S_{\mathrm{Bulk}}[\varphi]} \equiv \Psi[\bar{\varphi}] \\ e^{i W_{CFT}[\bar{\varphi}]} = \int D\psi\, e^{i S_{CFT}[\psi, \bar{\varphi}]} \end{cases}$$

$$\Psi[\bar{\varphi}] = e^{i W_{CFT}[\bar{\varphi}]}$$

$$ds^2 = a^2(\eta)[-d\eta^2 + d\mathbf{x}^2], \qquad a(\eta) = -1/H\eta$$

Gaussian wave functional

$$\Psi[\bar{\phi}(\mathbf{x})] \propto e^{\frac{i}{2}\ a^{d-1} \int d^d\mathbf{k}\ \frac{v'_{\mathbf{k}}}{v_{\mathbf{k}}} |\phi_{\mathbf{k}}|^2}.$$

$$\phi(\mathbf{x}) = \int d^d\mathbf{k} \frac{e^{i\mathbf{k}\mathbf{x}}}{(2\pi)^{d/2}} \phi_{\mathbf{k}},$$

$$v^*_{\mathbf{k}} v'_{\mathbf{k}} - v_{\mathbf{k}} v'^*_{\mathbf{k}} = i a^{1-d}$$

Choice of vacuum (for m=0)

$$v_{\mathbf{k}}(\eta) = \frac{\pi^{1/2}}{2} a^{-d/2} H^{(1)}_{d/2}(k\eta)$$

Bunch-Davies

(d+1=5)

$$\Psi[\bar{\phi}(\mathbf{x})] \propto \exp\left[\frac{i}{2} \int d^4\mathbf{k} \left(\frac{-H^{-1}k^2a^2}{2} + \underline{\frac{1}{8}}[i\pi + 2\gamma + \ln(k^2\eta^2)]k^4H^{-3} + O(a^{-2})\right) |\phi_{\mathbf{k}}|^2\right]$$

$$|\Psi|^2 \propto \exp\left[-\int d^4\mathbf{k}\left(\frac{\pi}{8}k^4H^{-3}\right)|\phi_{\mathbf{k}}|^2\right]$$

$$\langle\phi^*_{\mathbf{k}}\phi_{\mathbf{k}'}\rangle = \frac{8H^3}{\pi}k^{-4}\delta(\mathbf{k}'-\mathbf{k})$$

Real part of iW_{CFT} is all we use in cosmology.
But there is also the non-local part

$$W_{CFT} = \frac{H^{-3}}{16}\int d^4\mathbf{k}\ k^4\ln(k^2/H^2)\ |\phi_{\mathbf{k}}|^2 + \text{analytic.}$$

$$\langle T(k)T^*(k')\rangle \sim c\ k^4 \ln k^2$$

Expected form in a CFT

(As in Gubser, Klebanov, Polyakov 98)

Expected form in a weakly coupled CFT

$$W[g_{ij}] = \int d^4x \sqrt{g}\, \left[c_1 R \ln(\Box/\mu^2) R + c_2 R_{ij} \ln(\Box/\mu^2) R^{ij} + c_3 R_{ijkl} \ln(\Box/\mu^2) R^{ijkl}\right] + \text{analytic.}$$

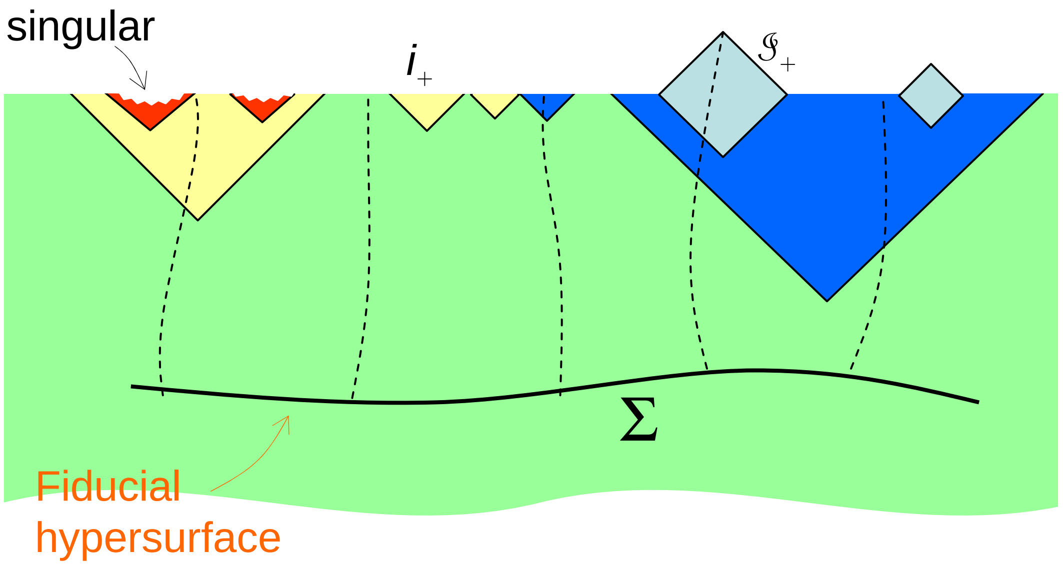
Coefficient of logarithmically divergent term is the trace anomaly

$$a_2 \sim \int d^4x \sqrt{g}\, \left[c_1 R^2 + c_2 R_{ij} R^{ij} + c_3 R_{ijkl} R^{ijkl}\right].$$

Proposal:

The boundary theory lives at the future boundary of the multiverse (suitably defined).

Future infinity



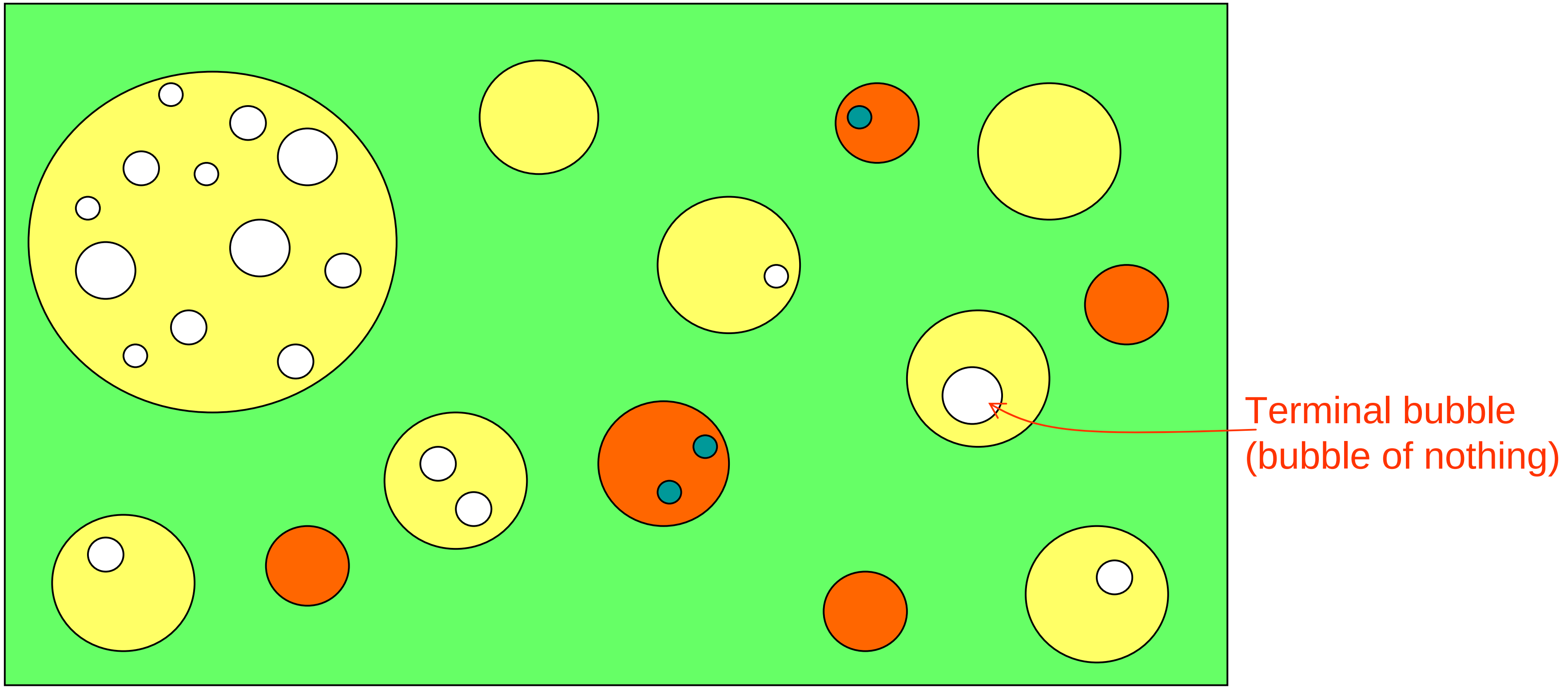
- Geodesic congruence projects bubbles onto Σ .
  $\Rightarrow$ Map of future infinity.
- Excise images of Minkowski bubbles. (They are described by the *2D* boundary degrees of freedom. *(FSSY)*
- AdS bubbles can be excised in a similar way (?).

(Horowitz, Hertog 05)

The future infinity F includes eternal points (and the boundaries of excised terminal bubbles).

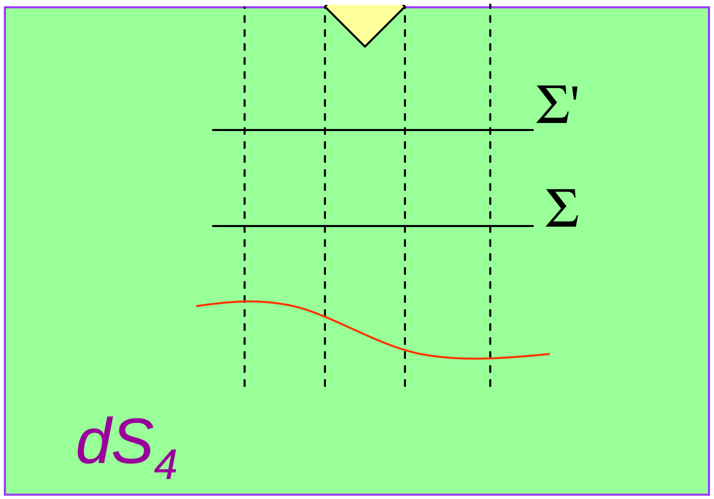
The metric $g_{ij}(\boldsymbol{x})$ on Σ defines a metric on F

Structure of the future boundary *F*



- Each bubble becomes a fractal “sponge” in the limit.
- Terminal bubbles correspond to holes (with 2D CFTs on their boundaries).
- Bubbles walls are sources in the boundary theory.

Different choices of Σ are related by Weyl rescalings.

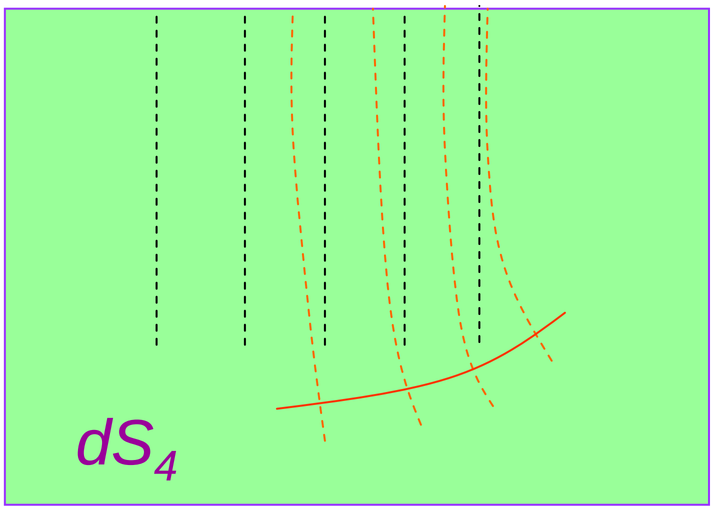


Move up along the congruence $\Rightarrow$ the comoving size of the bubble grows.

$$g_{ij}(\vec{x}) \rightarrow C g_{ij}(\vec{x}).$$

More generally,

$$g_{ij}(\vec{x}) \rightarrow f(\vec{x}) g_{ij}(\vec{x}).$$



All congruences become comoving asymptotically.

Conformal invariance of the boundary theory

- The scale factor measure is independent of the choice of fiducial hypersurface Σ in the limit $a \to \infty$.

- Different choices of Σ are related by scale transformations on *F* :

$$g_{ij}(\vec{x}) \to f(\vec{x}) g_{ij}(\vec{x}).$$

- $a \to \infty$ corresponds to UV limit on *F* : $\xi \to 0$.

This suggests that the boundary theory is scale-invariant in the UV. Some evidence:

- $\langle TT \rangle$ correlator has the form expected in a CFT.

- Distribution of nested bubbles is invariant under the Euclidean conformal group in the UV

Distribution of nested bubbles

Simplified model:

- System of nested bubbles. Neglect slow roll phase and tensor modes.
- Spacetime can be foliated with flat hypersurfaces
- Distribution of bubbles is scale invariant at late times (UV)
  (at early times there is the effect of initial conditions)

$$dN_{ij} = C_{ij}\, R^{-(3-q)}\, d\ln R$$

Transforms trivially under rescalings.

- Distribution of bubbles is invariant under
  Special Conformal Transformations in the UV.

$$\frac{x'^i}{\mathrm{x}'^2} = \frac{x^i}{\mathrm{x}^2} - b^i$$

BUBBLE FLUCTUATIONS

Bulk calculation leads to:

$$W[\bar{\delta}] = \frac{TR_0}{H^2}\sum_{LM}\frac{\Delta(\Delta+2)}{16}\left[\ln\left(\frac{R_0^2}{4\tilde{a}^2}\right)+2\left(\psi(L)+\frac{1}{L}\right)+i\pi+2\gamma\right]|\delta_{LM}|^2+\dots$$

Expected result in weakly coupled CFT

$$a_{3/2} = \int d\Sigma_2\left[d_1\left(K_{ab}K^{ab}-\frac{1}{2}K^2\right)+d_2\hat{R}\right] \propto \int d\Omega\ \delta\Delta(\Delta+2)\delta$$

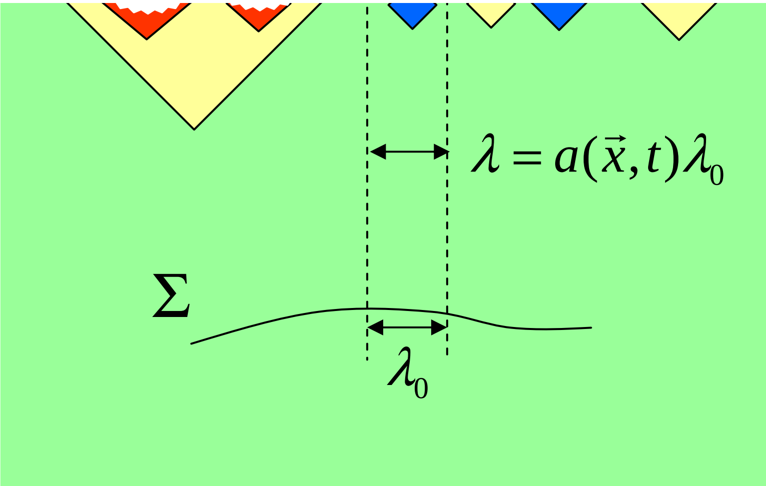
Boundary measure

Renormalization of boundary theory:

Integrate out short-wavelength modes $\lambda_0 < \xi$ ← **Boundary UV cutoff**

The corresponding 4D modes have minimum wavelength

$$\lambda_{\min}(\vec{x},t) = a(\vec{x},t)\,\xi.$$



Require $\lambda_{\min} < \ell$ ← Bulk resolution scale

$\Longrightarrow$ $a_{\max} = \ell / \xi$ -- scale factor cutoff

$$\xi \to 0 \quad \Rightarrow \quad a_{\max} \to \infty .$$

UV cutoff on the boundary ⟷ (IR) scale factor cutoff in 4D.

RG flow on the boundary ⟷ scale-factor time evolution.

Correspondence is accurate on super-horizon scales.

Conclusions

- The dynamics of the multiverse may be encoded in its future boundary *F*, in terms of a theory which is conformal in the UV.
- The measure can be obtained by imposing a UV cutoff in the boundary theory.
- This measure is closely related to the scale factor cutoff.
- The measure may perhaps be formulated even if there is no dual theory, based on the property of conformal invariance of the wave function.