Cosmological Perturbations In A New Type Of Chaplygin Gas

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Outline

1. Introduction
   - Inflation
   - Models
   - Cosmological Perturbations

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Early Universe

- Why Inflation?
  - Horizon problem
  - Flatness problem
  - ...
- A stage of accelerated expansion, $\ddot{a} > 0$, in the early universe.
- The Hubble radius, $(aH)^{-1}$, decreases in the inflationary phase.
- Observation: Inflation is supported by CMB data (WMAP, ...).

Outline

Modified Chaplygin Gas

Model: Inflation is driven by a modified Chaplygin Gas (Kamenshchik et al. 01, Bento et al. 02)

- The idea is to extend the Chaplygin Gas model to the early universe;
- How? Modified Chaplygin Gas that interpolates between inflationary era and a radiation dominated universe;
- Simplest option through a perfect fluid.

$$\boxed{\rho = \left(A + \frac{B}{a^{4(1+\alpha)}}\right)^{\frac{1}{1+\alpha}}}$$

$$\boxed{p = \frac{1}{3}\rho - \frac{4}{3}\frac{A}{\rho^{\alpha}}}$$

Modified Chaplygin Gas

Model: Inflation is driven by a modified Chaplygin Gas.

- $\rho = \left(A + \frac{B}{a^{4(1+\alpha)}}\right)^{\frac{1}{1+\alpha}} \quad , \quad 1 + \alpha < 0$
- $\rho = \left(A + B\, a^{4|1+\alpha|}\right)^{\frac{1}{1+\alpha}} \quad \Longrightarrow \quad \rho = A^{\frac{1}{1+\alpha}} \quad \vee \quad \rho = \frac{B^{\frac{1}{1+\alpha}}}{a^4}$
- $p = \frac{1}{3}\rho - \frac{4}{3}\,\frac{A}{\rho^{\alpha}} \quad \Longrightarrow \quad \rho + 3\,p < 0 \quad \Longrightarrow \quad \ddot{a} > 0$

Modified Chaplygin Gas - Analytical results

- The Friedmann equation, $H^2 = \frac{8\pi G}{3}\rho$, can be analytically integrated, involving hypergeometric functions.
  - In cosmic time:

$$2\sqrt{\frac{8\pi \mathrm{G}}{3}} A^{\frac{1}{2(1+\alpha)}} (t - t_\star) = (y+1)^r F\left(1, -r; 1-r; \frac{1}{1+y}\right) - F(-r, -r; 1-r; -1)$$

  - In conformal time:

$$a_\star A^{\frac{1}{2(1+\alpha)}} \sqrt{\frac{8\pi \mathrm{G}}{3}} (\eta - \eta_\star) = F\left(-r, -\frac{r}{2}; 1-\frac{r}{2}; -1\right) - y^{-\frac{r}{2}} (1+y)^r F\left(-r, 1; 1-\frac{r}{2}; \frac{y}{1+y}\right)$$

- Were $y = (a/a_\star)^{-4(1+\alpha)}$ and $a_\star$ is the end of inflation.
- The Friedmann equation doesn't have an exact solution and the scale factor is implicitly defined by these equations.

Background Equations I

Evolution of the universe:

- The Einstein's equations with the FRW metric in conformal time give:

$$\frac{a''}{a} = \frac{4\pi\,\mathrm{G}}{3}(\rho - 3p) \quad , \quad \frac{a'}{a} = \sqrt{\frac{8\pi\,\mathrm{G}}{3}}\rho^{1/2}$$

- For the gravitational wave power spectrum it is necessary to solve $X(\eta)$:

$$X'' + \left(k^2 - \frac{a''}{a}\right) X = 0$$

- The scale factor doesn´t have an explicit expression for this model, then the power spectrum must be calculated using numerical methods.

Background Equations II

Including Reheating through a description in terms of a scalar field ϕ.

- Inflationary dynamics: a homogeneous scalar field ϕ, drives inflation,

$$\rho = \frac{\phi'^2}{2\,a^2} + V(\phi) \quad , \quad p = \frac{\phi'^2}{2\,a^2} + V(\phi)$$

- The Klein-Gordon equation, including reheating through a transfer of energy between the scalar field and the radiation fluid, ρ_r, described by a phenomenological parameter Γ_ϕ, is:

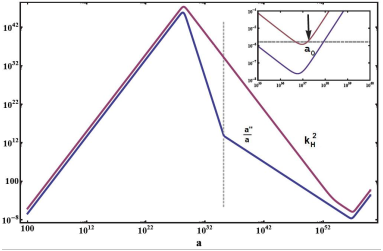
$$\phi'' + 2\frac{a'}{a}\phi' + a^2\,V'(\phi) = -\Gamma_\phi\,a\,\phi$$

- With the equation to the radiation fluid:

$$\rho_r' + 4\frac{a'}{a}\rho_r = \Gamma_\phi\frac{\phi'^2}{a}$$

Early Universe
Model 1

1. All the transition inflation-radiation is given by the Modified Chaplygin Gas for the early time;
2. For late time we use LCDM.



- The transition inflation-radiation is smooth, fact that will be clear in the gravitational wave power spectrum.

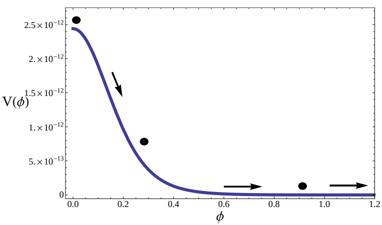
Early Universe
Model 2

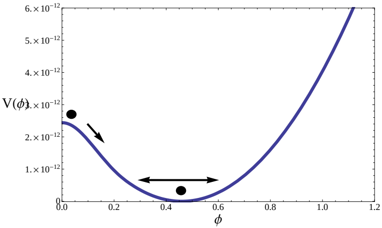
1. Until the end of inflation the equivalent scalar field potential for the modified Chaplygin Gas is

$$V_1(\phi) = \frac{V_{10}}{3}\left[\cosh^{\frac{2}{1+\alpha}}(-k(1+\alpha)\phi) + 2\ \cosh^{-\frac{2\alpha}{1+\alpha}}(-k(1+\alpha)\phi)\right]$$

2. Reheating through a potential of a quadratic form, leading to oscillations of the scalar field around the minimum of the potential ϕ_0:

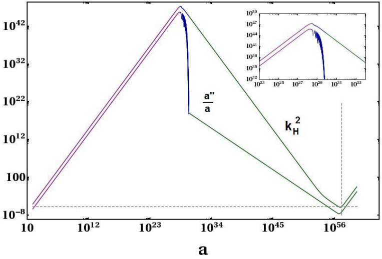
$$V_2(\phi) = V_{20}(\phi - \phi_0)^{2n}$$





Early Universe
Model 2

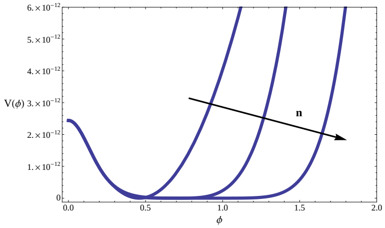
1. All the inflation is given by the Modified Chaplygin Gas for the early time.
2. Transition from inflation to radiation through a transfer of energy between the scalar field and the radiation fluid.
3. For late time we use LCDM.



The transition inflation-radiation has many oscillations, witch can imply significant variations in the high-frequency range of gravitational wave power spectrum.

Quadratic scalar field potential

- This model, $V_2(\phi) = V_{20}(\phi - \phi_0)^{2n}$, is important because of taylor-series expansions of scalar field potentials:



- The scalar field oscillates in turn of the minimum of the potential.
- Possible significant imprint in the high frequency range of the gravitational wave power spectrum.

Late Time

- To explain late time expansion through dark energy LCDM:

$$H^2 = H_0^2 \left[\Omega_r \left(\frac{a_0}{a}\right)^4 + \Omega_m \left(\frac{a_0}{a}\right)^3 + \Omega_\Lambda\right]$$

- Generalised Chaplygin Gas using two different forms:
  1. As a unified model:
  $$H^2 = H_0^2 \left[\Omega_r \left(\frac{a_0}{a}\right)^4 + \Omega_b \left(\frac{a_0}{a}\right)^3 + \frac{8\pi G}{3H_0^2}\left(A_2 + \frac{B_2}{(a/a_0)^{3(1+\alpha_2)}}\right)^{\frac{1}{1+\alpha_2}}\right]$$
  2. As a dark energy model:
  $$H^2 = H_0^2 \left[\Omega_r \left(\frac{a_0}{a}\right)^4 + \Omega_b \left(\frac{a_0}{a}\right)^3 + \Omega_{ch}\left(A_S + \frac{1-A_S}{(a/a_0)^{3(1+\alpha_2)}}\right)^{\frac{1}{1+\alpha_2}}\right]$$
- The power spectrum is not affected by α_2 parameter.

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Observational Constraints

Cosmological perturbations:

- The most general linear perturbation of the FRW metric can be expressed as

$$\mathrm{d}s^2 = a^2 \left\{ -(1 + 2\,A)\,\mathrm{d}\eta^2 + 2\,B_i\,\mathrm{d}x^i \mathrm{d}\eta + (\delta_{ij} + h_{ij})\mathrm{d}x^i \mathrm{d}x^j \right\}$$

Observational constraints:

- Density Perturbations
- Gravitational Waves

Cosmological Gravitational Waves

Quantum fluctuations in the early universe create a spectrum of gravitational waves.

- Perturbed metric of a flat universe, in conformal time η

$$ds^2 = a^2(\eta)\left\{-d\eta^2 + \left[\delta_{ij} + h_{ij}(\eta, x)\right] dx^i\, dx^j\right\}$$

- Tensorial perturbations h_{ij} expanded in plane waves

$$h_{ij}(\eta, \vec{x}) = \sqrt{8\pi \mathrm{G}} \sum_{\lambda=1}^{2} \int \frac{\mathrm{d}^3 k}{(2\pi)^{3/2} a(\eta)\sqrt{2k}} \left[\hat{a}_{\vec{k},\lambda}(\eta, \vec{k})\ \epsilon_{ij}(\vec{k}, p)\ e^{i\,\vec{k}\cdot\vec{x}}\ \xi(\eta, k) + \mathrm{h.c.}\right]$$

  Where $k = |\mathbf{k}| = 2\pi\, a/\lambda = a\,\omega$ is the comoving wave number.
  The annihilation and creation operators change with time.

- The mode function obeys the equation:

$$\xi'' + \left(k^2 - \frac{a''}{a}\right)\xi = 0$$

Gravitational Wave Power Spectrum

- Bogoliubov transformation to time-fixed annihilation operators through bogoliubov coefficients α and β:

$$\hat{a}_{\vec{k},\lambda} = \alpha(\eta, k)\, A_\lambda(\mathbf{k}) + \beta^*(\eta, k)\, A^\dagger_\lambda(\mathbf{k})$$

- The coefficient β gives the number of gravitons created in a given time η for a mode k, $|\beta|^2 = \langle N_K(\eta)\rangle$. The power spectrum is

$$P(\omega) = \frac{\hbar\omega^3}{\pi^2 c^3}|\beta|^2$$

- With $\rho_{\mathrm{GW}} = \int P(\omega)\, d\omega$, the dimensionless relative logarithmic energy spectrum at the present time η_0 is given by

$$\Omega_{\mathrm{GW}}(\omega, \eta_0) = \frac{1}{\rho_c(\eta_0)} \frac{\mathrm{d}\,\rho_{\mathrm{GW}}}{\mathrm{d}\,\ln\omega} = \frac{8\,\hbar\mathrm{G}}{3\,\pi\, c^5 H^2(\eta_0)}\,\omega^4\beta^2(\eta_0)$$

Bogoliubov Coefficient β

- The Bogoliubov coefficients obey the set of differential equations

$$\alpha' = \frac{i}{2\,k}\left[\alpha + \beta\, e^{2i\,k(\eta-\eta_i)}\right]\frac{a''}{a} \quad , \quad \beta' = -\frac{i}{2\,k}\left[\beta + \alpha, e^{-2i\,k(\eta-\eta_i)}\right]\frac{a''}{a}$$

- That, with the change of variables

$$\alpha = \frac{1}{2}(X+Y)e^{ik(\eta-\eta_i)} \quad , \quad \beta = \frac{1}{2}(X-Y)e^{ik(\eta-\eta_i)}$$

- And $X' = -i\,k\,Y$ becomes

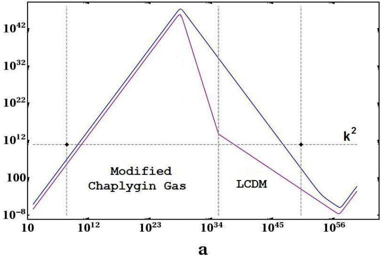
$$X'' + \left(k^2 - \frac{a''}{a}\right)X = 0$$

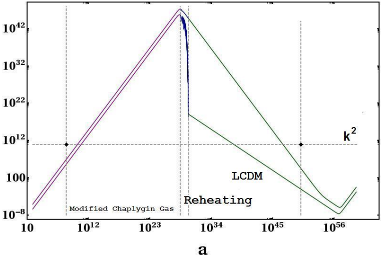
- For a de Sitter universe, within an inflationary period, $a(\eta) = -\frac{1}{H(\eta-\eta_1)}$, when H is approximately constant, $H \approx \frac{8\pi G}{3} A^{\frac{1}{1+\alpha}}$ for our model, has an exact solution,

$$X(\eta_i) = \left(1 + \frac{i\,a(\eta_i)\,H}{k}\right) e^{\frac{i\,k}{a(\eta_i)\,H}} \quad , \quad Y(\eta_i) = \left(1 + \frac{i\,a(\eta_i)\,H}{k} - \frac{a^2(\eta_i)\,H}{k}\right) e^{\frac{i\,k}{a(\eta_i)\,H}}$$

Numerical Integration

- We integrate from the early universe until late-time:

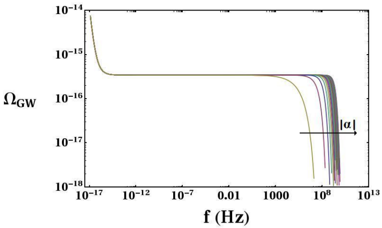




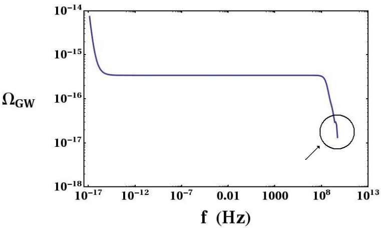
- The limits of integration are determined by the crossing of the horizon.

Results - Gravitational Wave Power Spectrum

Modified Chaplygin Gas



Including Reheating



Scalar Perturbations

Slow-roll aproximation:

- Power spectrum:

$$P_S(k) = \frac{128\pi}{3} \left. \frac{V(\phi)^3}{V'(\phi)^2} \right|_{\phi=\phi_c}$$

- Slow-roll parameters:

$$\epsilon = \frac{1}{16\pi} \left(\frac{V'(\phi)}{V(\phi)} \right)^2 \quad , \quad \eta = \frac{1}{8\pi} \frac{V''(\phi)}{V(\phi)} \quad , \quad \xi = \frac{1}{64\pi^2} \frac{V'(\phi)\, V'''(\phi)}{V(\phi)}$$

- Spectral parameters:

$$n_s = 1 - 6\epsilon + 2\,\eta \quad , \quad \alpha_s = 16\,\epsilon\,\eta - 24\,\epsilon^2 - 2\,\xi$$

- Experimental values for $k_c = 0.05\ \mathrm{Mpc}^{-1}$:

$$P_S(k_c) = (2.45 \pm 0.23) \times 10^{-9} \quad , \quad n_S(k_c) = 1.0 \pm 0.1 \quad , \quad |\alpha_S(k_c)| < 0.04$$

- Through this picture we can constrain the α value: ***Work in progress***.

Conclusions

- New model for the inflation to radiation transition
- We have included the reheating through a toy model
- *Work in Progress*: higher orders of the second potential in the second model
- *Work in Progress*: Scalar perturbations