Singularities in Loop Quantum Cosmology

Antonio Cardoso
University of Portsmouth

Thomas Cailleteau, Kevin Vandersloot, David Wands
Universite Paris-Sud 11, University of Portsmouth

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Introduction

- The absence of singularities may be taken to be a pre-requisite for any fundamental theory of nature
- In particular, we expect a quantum theory of gravity to resolve the curvature singularities found in Einstein's theory of general relativity (GR)
- Loop quantum cosmology (LQC) has been shown to cure the big bang singularity and replace it with a big bounce for Friedmann-Robertson-Walker (FRW) spacetimes
- The bounce occurs when the matter energy density reaches a Planckian value and quantum gravity effects behave repulsively
- We study both canonical and phantom scalar fields with exponential potentials using the effective Friedmann equations of LQC

Scalar field

- The energy density and pressure of a scalar field are given by

$$\rho = \pm\frac{1}{2}\dot{\phi}^2 + V \qquad p = \pm\frac{1}{2}\dot{\phi}^2 - V$$

- The plus sign indicates a canonical scalar field, and the minus corresponds to a phantom field
- The local conservation of energy-momentum leads to the Klein-Gordon equation governing the dynamics of the scalar field

$$\ddot{\phi} + 3H\dot{\phi} \pm \frac{\partial V(\phi)}{\partial \phi} = 0 \qquad H \equiv \dot{a}/a$$

- $V(\phi)$ represents the scalar field potential

Exponential potential

- We will consider exponential potentials of the form

$$V = V_0 e^{\lambda\kappa\phi} \qquad \kappa^2 = 8\pi G_N$$

- Without loss of generality, we take $\lambda > 0$
- Scalar fields with exponentials commonly arise in effective theories derived from dimensional reductions of higher dimensional models (e.g., string theory)
- They appear, for example, in the four-dimensional effective theory in the ekpyrotic scenario

  Khoury, Ovrut, Steinhardt, Turok (2001)
- We have found qualitatively similar results for simple polynomial potentials

Friedmann equations

- In GR, the Friedmann equations for a spatially flat FRW universe are given by

$$H^2 = \frac{\kappa^2}{3}\rho \qquad \dot{H} = -\frac{\kappa^2}{2}(\rho + p)$$

- Canonical scalar field with an exponential potential: big bang singularity, where $H \to \infty$ as $a \to 0$ at a finite proper time in the past (and/or a big crunch singularity in the future)
- Phantom scalar field with a positive exponential potential ($V_0 > 0$): big rip singularity, where $H \to \infty$ and $a \to \infty$ at a finite proper time in the future (or the time reverse in the past)
- Note that one cannot have a phantom scalar field with a negative potential, as the total energy density must be non-negative in a spatially flat FRW cosmology

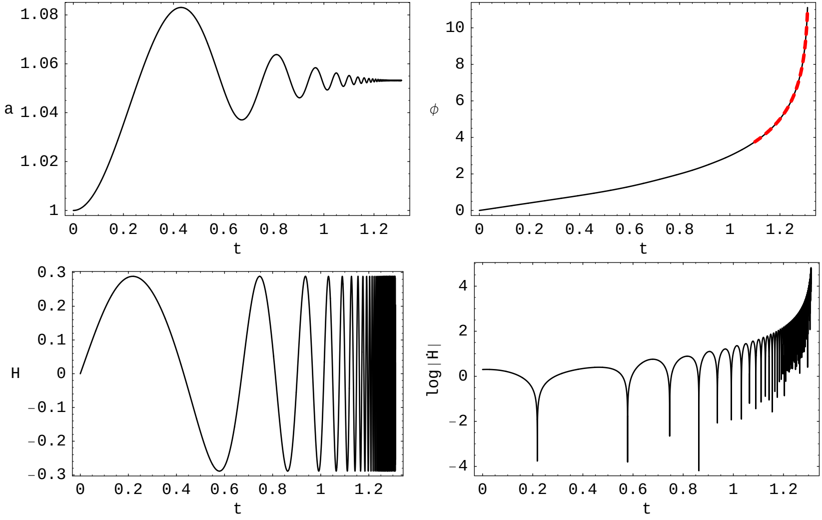
Effective Friedmann equations

- To incorporate the quantum effects due to LQC, we consider effective Friedmann equations with corrections of the form
  Ashtekar, Pawlowski, Singh (2006)

$$H^2 = \frac{\kappa^2 \rho}{3}\left[1 - \frac{\rho}{\rho_c}\right] \quad \dot{H} = -\frac{\kappa^2 (\rho + p)}{2}\left[1 - 2\frac{\rho}{\rho_c}\right] \quad \rho_c \sim \kappa^{-4}$$

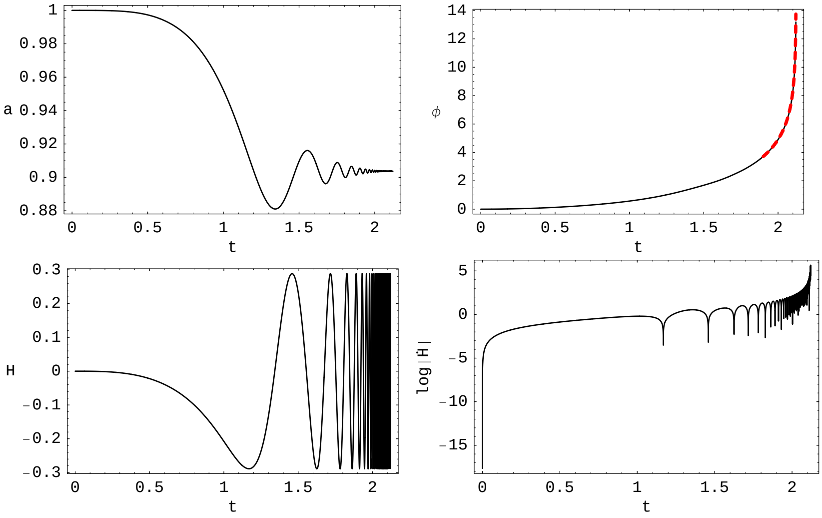
- These equations are approximations to the true quantum dynamics
- The classical GR limit can be recovered by letting $\rho_c \to \infty$
- The loop effects do not modify the form of the scalar field equations, and thus the Klein-Gordon equation is unchanged
- Canonical scalar field with a positive exponential potential ($V_0 > 0$): the past big bang or future big crunch singularity of the GR solutions is replaced by a bounce, and the cosmology becomes non-singular

Numeric results: canonical scalar field



$\lambda = 1 \quad V_0 = -\kappa^{-4} \quad \rho_c = \kappa^{-4} \quad (\kappa = 1)$

Numeric results: phantom scalar field



$\lambda = 1 \quad V_0 = \kappa^{-4} \quad \rho_c = \kappa^{-4} \quad (\kappa = 1)$

Numeric results: comments

- The qualitative results do not depend on the choice of initial conditions
- The solutions have a “quiescent” or “sudden” singularity, where a and H are both bounded but $\dot{H}$ diverges
  Shtanov, Sahni (2002); Barrow (2004)
- The Ricci curvature scalar $(R = 6\dot{H} + 12H^2)$ diverges at a finite proper time
- Note that, although the curvature diverges, these singularities are weak in that they may be geodesically complete
  Fernandez-Jambrina, Lazkoz (2004); Singh (2009)
- In order to verify that the singular nature of the solutions is not a numerical artifact, we can find an approximate analytical solution for the regime where $a(t)$ is approximately constant

Unbounded potential

- The effective Friedmann equation requires that ρ is bounded, and for a canonical field the upper bound $\rho \leq \rho_c$ also requires that $V \leq \rho_c$, but the potential is not bounded from below
- We can write the evolution equation as

$$\dot{H} = -\kappa^2(\rho - V)\left[1 - 2\frac{\rho}{\rho_c}\right]$$

- Thus, for a canonical field $\dot{H}$ is unbounded only if $V(\phi)$ is unbounded from below
- Similarly, for a phantom field one can show that $\dot{H}$ is unbounded only if $V(\phi)$ is unbounded from above
- If we modify the exponential potential to introduce a lower (upper) bound, the sudden singularity disappears

Double exponential potential

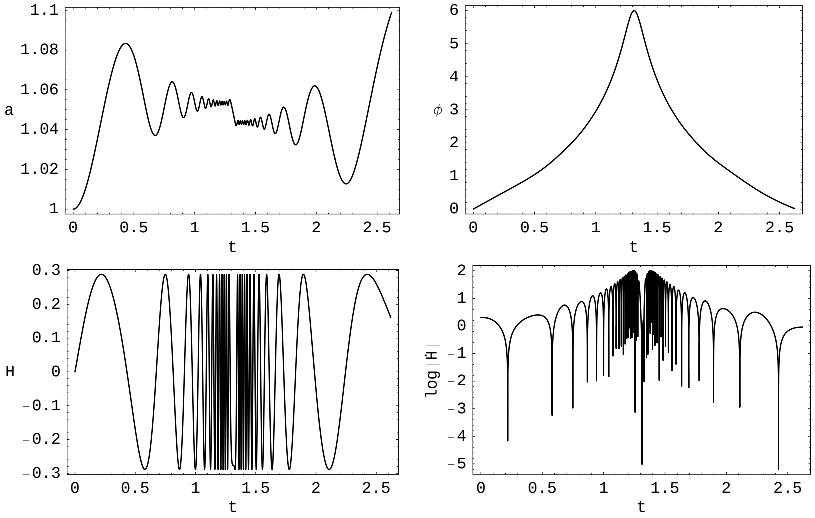
- Consider a canonical scalar field with a potential of the form

$$V(\phi) = V_0 e^{\lambda\kappa\phi} + V_2 e^{\lambda_2\kappa\phi}$$

- We choose $V_0 < 0$ and $\lambda > 0$, as before, but $V_2 > 0$ and $\lambda_2 > \lambda$
- These conditions ensure that the potential is bounded from below and grows as a positive exponential for large values of ϕ
- In the numerical run, the parameter values were chosen to be $V_0 = -\kappa^{-4}$, $\lambda = 1$, $V_2 = e^{-6}\kappa^{-4}$, and $\lambda_2 = 2$
- The potential minimum is located at

$$\phi_{min} = \frac{1}{\kappa(\lambda_2 - \lambda)} \ln\left(-\frac{V_0\lambda}{V_2\lambda_2}\right) \approx 5.3\kappa^{-1}$$

Numerical results: canonical scalar field



$V_0 = -\kappa^{-4} \quad \lambda = 1 \quad V_2 = e^{-6}\kappa^{-4} \quad \lambda_2 = 2 \quad (\kappa = 1)$

Numeric results: comments

- We have chosen the same initial conditions as for the single exponential case
- The solution is non-singular, as is shown by the boundedness of both the Hubble rate H and its time derivative $\dot{H}$
- The scalar field passes through the potential minimum and rolls up the potential only to turn around at a point where the potential is positive, which is roughly given by $\phi \approx 6\kappa^{-1}$
- Similar behavior occurs in the phantom case for a double exponential potential with $V_0 > 0$ and $V_2 < 0$, where the potential is now bounded from above, and the behavior is also non-singular

Conclusions

- We have shown that singular solutions exist to the LQC effective Friedmann equations for potentials that are not bounded from below (above) for a canonical (phantom) scalar field
- It is interesting that sudden singularities, which may appear to be somewhat contrived in GR, appear in quite simple models in LQC
- While this may indicate that generic singularity resolution is not a feature of loop quantum gravity, it may as well simply imply that the effective equations of LQC break-down in some specific cases
- The scalar field Lagrangians which give rise to singular behavior may also themselves be regarded as sufficiently pathological that this need not be considered as a significant limitation of LQC